

1.1 – What is Mathematical Modeling?

1 Mathematical Modeling

There are many definitions of mathematical modeling, since it became its own topic of study in the late nineteenth century [5]. Some discuss mathematical modeling as the model itself, the construct or tool used to solve a problem [2]. Others discuss mathematical modeling as a process used to analyze complex real-world problems and provide insight [4]. For the purposes of this course we are going to use the following definition:

Definition 1.1.1 (Mathematical Modeling)

Mathematical modeling is a tool, a process and a mindset used to represent, analyze, and make predictions about real-world systems or phenomena.

While it is true that mathematical modeling uses tools, including the various types of models introduced in this course, modeling is not just knowing and using the tools. Modeling encompasses a problem solving process that helps turn a real-world problem into a solvable mathematical problem, which can then be interpreted back into real-world terms. We will discuss the process we use throughout the course in detail later, but the modeling process helps us think logically through a problem and arrive at a data-driven solution [1]. Finally, modeling is a mindset [2]. It requires creativity and active participation from the modeler – you! There is not always a “right” answer to a modeling problem, and it is up to you to make and defend choices. As we move through the course, the hope is that this logical way of thinking becomes a part of your natural thought process.

2 The Value of Modeling

Leaders are problem solvers. At the United States Military Academy (USMA) the overarching academic goal as defined in the Redbook is that “graduates integrate knowledge and skills from a variety of disciplines to anticipate and respond appropriately to opportunities and challenges in a changing world.” USMA expects its graduates to enter the U.S. Army as leaders who are able to communicate, think critically and creatively, and apply ethical frameworks to support their decisions [6]. The process and mindset of mathematical modeling imbue leaders with the ability to do all these things, regardless of if the problem requires mathematics to solve [1]. For example, a platoon leader is unlikely to be asked to factor polynomials or solve systems of equations by hand, but will be asked to provide solutions to ill defined problems that are not formulaic [1]. Learning and applying the concept of modeling is essential to success in both mathematics and the Army.

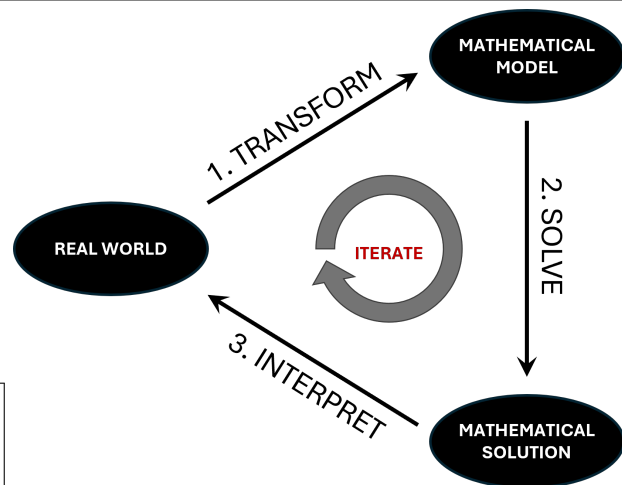


Figure 1: The mathematical modeling triangle, showing the process of *transforming* the real problem into mathematics, *solving* the defined model, and *interpreting* it back into real terms.

3 Mathematical Modeling Process

The mathematical modeling process used throughout this course is called the modeling triangle, as shown in Figure 1. The process has three steps: transform, solve, and interpret.

1. **Step 1. Transform:** The transform step takes the real-world problem and turns it into a mathematical model that can be solved. This is often the most challenging part of modeling and, arguably, the most important. If we select an inappropriate model, our ability to describe phenomena, predict behavior, and inform decisions is invalid. Transforming the real-world problem requires understanding what is *given* in the problem and what we need to *find*. These are the two main inputs for this step. Then we need to *explore* the problem. We need to identify the type of model to use, define variables, and make assumptions, see Definition 1.1.2 below. These three items don’t necessarily happen linearly and often need to be revisited throughout this step as more information is gained or as the problem is refined. The last component of the transform step is to clearly write out your *generalized model*, including your variable definitions, model assumptions, and the general model itself. The components that make up the transform step and a short definition of each are found in Figure 2.

Definition 1.1.2 (Assumptions)

Deliberate simplifications or idealizations of a real-world situation made in order to create a workable model [3].

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Transform Step [1].

1. **Given** - What does the problem say?
2. **Find** - What question(s) are you trying to answer?
3. **Explore** -
 - Define Variables - What values change?
 - Make Assumptions - What makes the problem simpler? What do you not know that you need to solve? What idealizations are needed to represent the real-world in the mathematical model you chose?
 - Model Type - How can you express the problem with math?
4. **Final Model** - Write the generalized model clearly.

- What assumptions to make.
- What relationship, equation, or function to use.
- What data to use or exclude.

Effective modeling decisions must be *reasonable* and *necessary*. For example, when developing a model to predict population growth we might assume an exponential model based on our exploration of the data and any prior knowledge of population growth models. This would be *reasonable* based on our exploration and *necessary* in order to create a model type and have a general model form [1]. Modeling decisions ultimately shape the structure, accuracy, and applicability of the final model.

Mathematical modeling is often interdisciplinary in nature. You may be asked to analyze situations from domains you are unfamiliar with, such as logistics, health care, or environmental science. In these cases, it is not only acceptable but expected that you consult subject matter experts. These conversations provide the essential context needed to make informed and responsible modeling decisions, especially when identifying key variables, choosing reasonable assumptions, or interpreting domain-specific data.

Modeling decisions inherently produce limitations because they simplify or abstract the real world. Modeling decisions reduce the complexity so we can create a model to solve. Understanding and clearly communicating model limitations are essential, as decisions made can invalidate the model for use in certain applications.

Figure 2: Transform step with questions to use as a quick reference.

2. **Step 2. Solve:** After developing the model, we can apply the most appropriate solution technique to find an answer. The solve step includes determining the numerical parameters to use in the general model to develop the specific model we are going to use to solve. As with model selection, the choice of solution technique can lead to suitable and unsuitable answers.
3. **Step 3. Interpret:** The solution is still in mathematical terms. We need to translate the solution back into the real world. Determine what the answer means in the context of the problem and translate it into simple, non-mathematical language for the decision-maker.

Iterative Process: The mathematical modeling process has a cyclical design with an iterative process. As a good Soldier continuously improves a defensive position, so a good problem solver continuously makes improvements to a mathematical model. In the interpret step we analyze our solution to assess whether it makes sense and solves the problem posed. If it does not, or reveals an additional problem to solve, we need to adapt our model by making iterative changes. We also need to ask how closely our mathematical model reflects the real world. We can assess the model's sensitivity to our assumptions. Our analysis will determine what adjustments we can make to develop a better model.

Definition 1.1.3 (Limitation)

A known boundary in a model that results from simplifying assumptions, incomplete data, or the chosen structure of the model, which affects the model's accuracy, applicability, or generalizability. [3, 4].

4 Modeling Decisions

A modeling decision is a deliberate choice made by the modeler during the transform step of the modeling process. Possible modeling decisions may include:

- What variables to incorporate.

5 Ethical Checklist

For the purposes of this course, we are going to focus on considering three things:

- Validity and implications of the data.
- Validity of the model and analysis.
- How the data and model are communicated.

These three ideas reflect core ethical standards outlined by the American Statistical Association for responsible data analysis and communication. This organization defines ethical practices to help statisticians make ethical decisions when they work with and communicate data. Although this is not a statistics course, the core ethical themes presented mirror those explored in MA206, but they are simplified to be more accessible to you as you learn modeling.

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The full ethics checklist expected to be used throughout the course is below:

- Data.** I have considered the potential implications of my data.

This could include, and is not limited to, the source of the data, who owns the data, the reliability and accuracy of the data, the method of data collection, the method(s) of cleaning the data.

- Analysis and Modeling.** I have considered the potential implications of my modeling decisions, the analysis, and my interpretation of the results.

This could include, and is not limited to, the ability of the selected model to answer the question that was asked, how well the model reflects the story of the data and/or the context of the problem, how my assumptions could have impacted my results, whether a robust solution is required, and how the results could impact stakeholders.

- Communication.** I have considered the potential implications of my communication of my findings.

This could include, and is not limited to, how well the visualizations support the analysis without being misleading, how any uncertainties are addressed in the communication, and how clearly the modeling decisions are explained and justified to an appropriate audience, such as a decision maker or other stakeholder.

If we have done our due diligence in considering these three aspects, then the model may be considered ethically sound. We will further explore this checklist throughout the course.

References

- [1] US Military Academy. *Modeling in a Real and Complex World*. West Point, New York: Department of Mathematical Sciences, 2022.
- [2] Alyssa Andreadis. *What Is Mathematical Modeling, Really?* COMAP, Apr. 2024. URL: <https://www.comap.com/blog/item/what-is-mathematical-modeling>.
- [3] K Bliss, K Fowler, and B Galluzzo. *Math Modeling*. 2014. (Visited on 05/10/2025).
- [4] S Garfunkel and M Montgomery. “GAIMME: Guidelines for assessment and instruction in mathematical modeling education”. In: *Bedford, MA, USA* (2019).
- [5] Hermann Schichl. “Models and the History of Modeling”. In: *Applied optimization* (Jan. 2004), pp. 25–36. DOI: [10.1007/978-1-4613-0215-5_2](https://doi.org/10.1007/978-1-4613-0215-5_2).
- [6] US Military Academy, Office of the Dean. *Academic Program: Curriculum and Course Descriptions*. <https://courses.westpoint.edu/static/index.htm#t=AcademicProgram.htm> Accessed: May 2025. 2025.