Background: How to Build a Linear First-Principles Model

For a linear model, $y = mx + b$, we only have two parameters, so we only have three questions to ask.

1. What is the value of the dependent variable when the independent variable is 0? (E.g., what is the population at time 0?) This gives us b.
2. Do we expect things to increase linearly ($m > 0$), decrease linearly ($m < 0$), or stay the same ($m = 0$)?
3. How quickly do we expect things to change? (This will give us the value of the slope, m.)

Task 1: Develop a First-Principles Model

Read the following scenarios and develop a first principle's model that gives the current population P, at time t, for Counterlandia and Flipsylvania.

Counterlandia

Suppose we are studying the population of a very tiny country of Counterlandia. When we start our study ($t = 0$), there are 10 residents. Each month, 6 more math counters try to immigrate into the country – they attempt entry by being flipped at the border. If a counter lands yellow-side up, the are allowed to enter; if a counter lands red-side up, the are not. Let's develop a first principle's model for this example.

$$P = \_\_\_\_\_t + \_\_\_\_\_$$

Flipsylvania

Suppose we are studying the population of another very tiny country of Flipsylvania. When we start our study ($t = 0$), there are 40 residents. Each month, 8 more math counters try to emigrate out of the country – they attempt departure by being flipped at the border. If a counter lands red-side up, they are allowed to leave; if it lands yellow-side up, they must stay. Let's develop a first principle's model for this example.

$$P = \_\_\_\_\_t + \_\_\_\_\_$$

Task 2: Develop an Empirical Model

In a group of three cadets, choose one of the scenarios above and conduct an experiment to develop an empirical model that gives the current population P, at time t, for Counterlandia or Flipsylvania. Following the immigration / emigration attempt procedures previously described (being tossed at the border and deciding based on whether you see the letter or not), determine what the population is after 7 months. Complete the following table to help you collect your experimental data.

My scenario is (immigration / emigration) in _____. My initial population is _____, and each month _____ individuals try to cross the border.

Time (months)	Successful Im/Em	Population
0	0	
1		
2		
3		
4		
5		
6		
7		

Fitting a Line

Once you have completed collecting your data, fit your data in Excel with a linear trendline, and record the equation of this model below. Write your first-principles model for comparison.

Empirical Model: $P =$ _____$t +$ _____

First-principles Model: $P =$ _____$t +$ _____

Approximating with Your Models

1. Using your empirical model, what do you expect the population to be after 10 months?

2. Using your first-principles model, what do you expect the population to be after 10 months?

3. Enable the R^2 value for your trendline in Excel for your empirical model. What is it? What does this number mean? Use a reliable online source to help you explain it in your own words.

4. Is a higher R^2 value always better? What are some reasons a model with a high R^2 might still be misleading or inappropriate for the data?

5. Compare the R^2 value from your empirical model to how well your first-principles model fits the data. Do they match? If not, what might explain the difference?