# First-Principles Models - Exponential Functions

For an exponential model, $y = ab^x$ or $y = a(1 + r)^x$, as with a linear model, we only have two parameters to determine, but the questions we ask are different because the rate of change is not constant. We need to ask the following questions:

1. What is the value of the dependent variable when the independent variable is 0? (E.g., what is the population at time 0?) This gives us $a$.
2. Do we expect the quantity to grow over time ($r > 0$) or to decay over time ($r < 0$)?
3. By what rate do we expect the quantity to grow or decay? This gives us the value of the growth or decay rate, $r$.

## Task 1: Develop a First-Principles Model

Read the following scenarios and develop a first principle's model that gives the current population $y$, at time $x$, for Countville and Flip Hollow.

### The Magical City of Countville

The city of Countville is a magical place. Countville originally starts with a population of 4 math counters. Every cycle (we don't know the length of the cycles in human time, because, well it is a magical city...), each member of the population tries to generate a new counter. The process works like this: flip the resident counters. If a counter lands yellow-side up, the counter stork instantly delivers a brand-new resident. If it lands red-side up, no new counter is born that cycle.

$$y = \_\_\_\_\_ \cdot \underline{(\quad\quad)}^x$$

### The Legend of Flip Hollow

Flip Hollow is a haunted place, steeped in legend. Each night, every resident is at risk of being offed by the Headless Counterman who searches for the yellow-side-up counters. If a counter is found yellow-side up, that resident vanishes; if it lands red-side up, they survive the night. To simulate a night in Flip Hollow, flip the counters, record the number that are yellow-side up, and remove these from your population. The initial population of Flip Hollow is 60 residents.

$$y = \_\_\_\_\_ \cdot \underline{(\quad\quad)}^x$$

## Task 2: Develop an Empirical Model - Birth and Death

In a group of three cadets, choose one of the scenarios above and conduct an experiment to develop an empirical model that gives the current population $y$, at time $x$, for Countville and Flip Hollow. Following the birth / death procedures previously described (being tossed and deciding based on whether pennies land heads-up or not), determine what the population is after 6 cycles. Complete the following table to help you collect your experimental data.

**My scenario is ( birth / death ) in ( Countville / Flip Hollow ). My initial population is _____.**

| Time (cycles) | Number of Births / Deaths | Population |
|---|---|---|
| 0 | 0 | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

### Fitting a Trend

Once you have completed collecting your data, fit your data in Excel with a trendline.

1. Try fitting the data with a linear trendline. Is this a good fit? Why or why not?

   Linear Trendline equation: _____

2. Try fitting the data with a exponential trendline. Is this a good fit? Why or why not?

   Exponential Trendline equation: _____

3. From a first-principles perspective, why might a linear trendline not be a great fit?

## Task 3: Compare Models

1. Using your empirical linear model, what do you expect the population to be after 7 cycles?

2. Using your empirical exponential model, what do you expect the population to be after 7 cycles?

3. Using your first-principles model, what do you expect the population to be after 7 cycles?

4. Compare your empirical exponential model and your first-principles model. Did you expect them to be more similar, why or why not?

5. Calculate the growth or decay rate, $r$, from your empirical model. How does this value compare to the $r$ value you used in the first-principles model?

6. Which model is better, empirical or first-principles? Explain why.