## Reading: Hillier & Hillier p. 375-380

## Problem

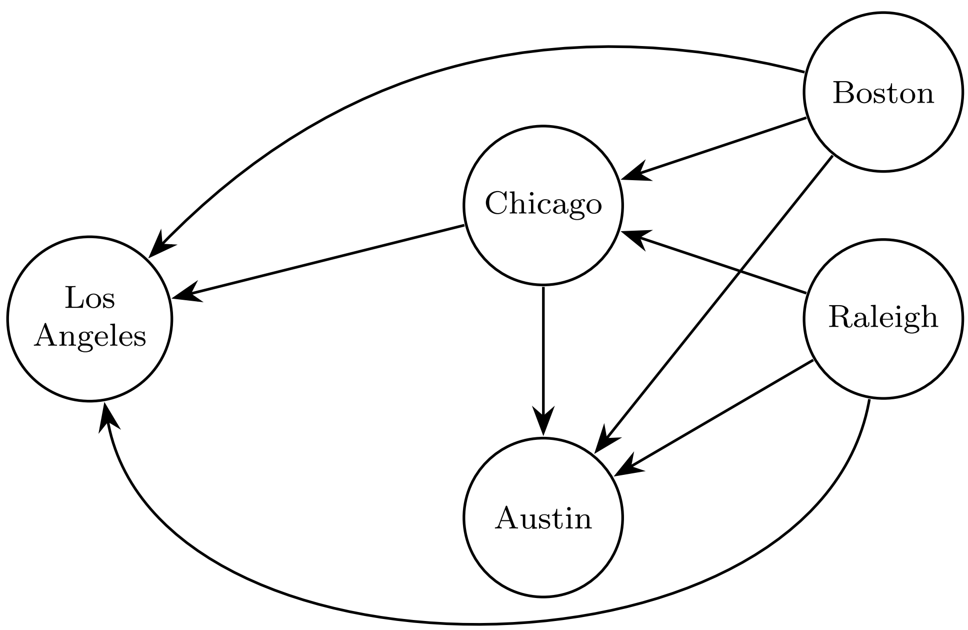
Each year, Data Corporal produces 400 computers in Boston and 300 computers in Raleigh. Los Angeles customers must receive 400 computers, and 300 computers must be supplied to Austin customers. Computers are transported by plane and may be sent through Chicago. However, flights from Boston to Chicago are limited to 65 computers and flights from Raleigh to Chicago are limited to 80 computers. The costs of shipping a computer between cities are shown in the table below.

| | To [$ per computer] | | |
|---|---|---|---|
| From | Chicago | Austin | Los Angeles |
| Boston | 80 | 220 | 280 |
| Raleigh | 100 | 140 | 170 |
| Chicago | - | 40 | 50 |

## Task 1

Formulate and solve a minimum-cost flow problem that can be used to minimize the cost of meeting Data Corporal's annual demand.

1. First visualize the distribution network using nodes to represent each city and arcs to represent routes between cities. Complete the network below by adding the applicable supply, demand, cost, and capacity to each node or arc. Reminder from the reading: by convention we write supply values as positive and demand values as negative.



2. Now that we've visualized the network and the given information, let's define our variables.

   We will use city name abbreviations for our set of **nodes**:
   $\mathcal{N} = \{B\,(Boston),\ R\,(Raleigh),\ C\,(Chicago),\ A\,(Austin),\ L\,(LosAngeles)\}$

   The possible shipping routes between cities then become our set of **arcs**:
   $A = \{(B,C),\ (B,A),\ (B,L),\ (R,C),\ (R,A),\ (R,L),\ (C,A),\ (C,L)\}$

   We can now define the following:
   $x_{ij}$ to be the number of computers shipped from city $i$ to city $j$,
   $c_{ij}$ to be the cost of shipping from city $i$ to city $j$, and
   $b_i$ to be the supply or demand at each node $i$.

3. Which of the terms defined above represent our **decision variables**?

4. Now that we've defined our variables, we can formulate our linear program. We'll start with the objective function:

$$\min \sum_{(i,j)\in\mathcal{A}} c_{ij}\,x_{ij}$$

   Here we want to minimize the total cost of shipment. We write the total cost as the sum of the cost per computer on each route, $c_{ij}$, multiplied by the number of computers we ship along that route, $x_{ij}$. Summation notation provides us a compact way to represent this sum. Use the space below to write the objective function using the decision variables and associated costs.

5. Let's move on to our constraints. Network problems share a common *flow balance* constraint which requires that flow through a node equal its supply or demand requirement. We write this flow conservation constraint using summation notation as follows:

$$\text{s.t.} \quad \sum_{j:\,(i,j)\in\mathcal{A}} x_{ij} \;-\; \sum_{k:\,(k,i)\in\mathcal{A}} x_{ki} \;=\; b_i \qquad \forall i\in\mathcal{N}$$

   (a) This notation tells us that the sum of all flow out of the node minus flow into the node must equal the supply or demand at each node in our set $\mathcal{N}$. Use the space below to write out the individual flow balance constraints for each node.

(b) Many network problems also have *capacity constraints* of the form,

$$L_{ij} \leq x_{ij} \leq U_{ij},$$

where $L_{ij}$ is the lower bound on flow through arc $(i, j)$ and $U_{ij}$ is the upper bound on flow through arc $(i, j)$. These constraints limit the amount of flow on the arcs. In this problem we only have two. Use the space below to write the capacity constraints:

(c) The last constraints we must consider in our network problem are the *non-negativity constraints*, $x_{ij} \geq 0$, which are common to many linear programs. Write the non-negativity constraints in the space below:

6. Now that we've properly formulated our linear program, we can solve to find the minimum shipping cost. Open the Excel file `MA103 PSL7 Network Optimization`. The technical coefficient matrix $A$ and the right hand side vector $\vec{b}$ are provided.

(a) What do you notice about the constraint matrix?

(b) Fill in the cost vector $\vec{c}$ and use `=MMULT()` to calculate the Total Cost.

(c) Use `=MMULT()` to calculate $A\vec{x}$.

(d) Use Solver to find the minimum shipping cost. Interpret the value of your decision variables in the space below:

## Challenge

1. How does the original formulation change if the specific arc capacity constraints into Chicago are removed and instead the requirement states that at most 200 units could be shipped through Chicago? Rewrite the new formulation and solve.

2. How does the original formulation change if Boston and Raleigh can produce up to 500 computers each? Rewrite the new formulation and solve.