MA103: Mathematical Modeling
Linear Functions I

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1. Understand the parameters of a linear model, slope and y-intercept, and the roles of these parameters in changing the shape of the model
2. Be able to interpret the meaning of the parameters of a given linear model in the context of a real world scenario
3. Build a linear model using a first-principles approach
4. Build a feasible linear model using an empirical approach

Review:

1. What is the slope-intercept form of a linear function?

2. What does the parameter m represent? What does it do to the shape of the function?

3. What does the parameter b represent? What does it do to the shape of the function?

Example Problem

You're helping the regional transit authority estimate pricing for local train tickets. Each ticket includes a base boarding fee and a fee per stop. While the fee structure is intended to be consistent, some variation occurs depending on time of day, kiosk rounding, and promotional discounts.

The data on the next page was collected from 5 travelers who purchased tickets at different times throughout the day. All were riding the same regional line, and the number of stops on their route is shown. Prices were rounded to the nearest \$0.05.

We know the base price for a ticket is \$2.50 and each stop adds \$1.50 to the ticket price. Develop a first-principles model for the price of a ticket based on the number of stops. Then use your model to determine how much it would take someone to travel 10 stops.

Now, using the data from the ticket prices build a model using an empirical approach. Use the empirical model to determine how much it would take someone to travel 10 stops.

Number of Stops	Ticket Price ($)
2	5.40
3	6.75
5	9.15
7	12.80
11	18.25

Which model is best to use?

Practice Problems

1. CDT Knight is testing a new compact oven and is curious about how fast it heats food. He places a slice of frozen pizza in the oven and uses a food thermometer to record the internal temperature every few minutes. However, because the pizza is not uniformly thick and the thermometer is hand-held, some slight variation occurs. The manufacturers of the oven claim that it can heat food at a rate of 10°C per minute. Develop a first-principles model based on this problem.

The data below shows the temperature readings (in °C) over time. Despite the noise, the temperature seems to be increasing steadily. Develop an empirical model for how the pizza's temperature changes over time. Use your model to estimate when the pizza reaches 100°C. Does oven manufacturer make an accurate claim?

Time (minutes)	Temperature (°C)
1	3.1
4	26.3
7	49.0
10	70.8
13	91.4

2. A wildlife research team is monitoring the digging habits of Mr. Mole using a collar with depth sensors. Mr. Mole was released at the surface and dug steadily downward toward his burrow at a rate of 2 meters per minute. Use your model to estimate when he reaches a burrow located at –40 meters.

The following table shows the reported depth (in meters below ground level) over time. The sensors take readings every few minutes, but the surrounding soil density and tunnel angles introduce slight inconsistencies in the reported altitudes. Develop an empirical model using the data and predict when he reaches his burrow.

Time (minutes)	Depth (m)
5	-18.7
6.5	-21.8
8	-24.9
9.5	-28.8
11	-33.5

3. During a mechanical test, engineers monitored how an engine's surface temperature changed as they increased its rotation speed. According to the manufacturer, the engine temperature should increase at no more than 0.8° per unit increase in rotation speed. The initial temperature of the engine is 14°. Develop a first-principles model for how temperature changes with rotation speed. Use your model to predict the engine temperature at 17 cycles per second.

The table shows the engine's rotation speed (in cycles per second) and the corresponding recorded temperature (in °C). Although the engine runs in a controlled environment, airflow variations and friction levels created slight inconsistencies in the temperature readings. The measurements were recorded during a single warm-up run. Then develop an empirical model using the data and make the same prediction.

Rotation Speed (cps)	Temperature (°C)
11	24.1
12	24.9
13	25.7
14	26.4
15	27.2

4. Think about how reasonable your models are. For example, in Problem 2, does it make sense that Mr. Mole keeps digging deeper forever? Why or why not? What does this tell you about when a model is useful and when it might break down?