MA103: Mathematical Modeling
Exponential Functions I **30 September 2025**

1. Understand the parameters of an exponential model, growth rate and y-intercept, and the roles of these parameters in changing the shape of the model
2. Be able to interpret the meaning of the parameters of a given exponential model in the context of a real world scenario
3. Build an exponential model using a first principles approach
4. Think through ethical considerations for using a linear model to make predictions

Review:

1. What is the general form of an exponential function?

2. What does the parameter a represent? What does it do to the shape of the function?

3. What does the parameter b represent? What does it do to the shape of the function?

Example Problem: Modeling Anti-Depressants in Blood Stream

After taking a single dose of a commonly prescribed antidepressant, it's important to allow enough time for the body to clear the medication before beginning a new treatment. Overlapping drugs like this can be dangerous.

You are a pharmacist consulting with a therapist on a patient who needs to switch medications. The patient has recently taken a single dose of a commonly prescribed antidepressant. Based on research, the body metabolizes and eliminates about 25% of this medication every 24 hours, meaning 75% of the dose remains each day.

The lab reports that the patient currently has 60 mg of the drug in their bloodstream. To safely start the new medication, the level must fall below 10 mg. Develop a model for the amount of medication remaining in the bloodstream and then use your model to determine the earliest day on which the patient can safely begin the new prescription.

Example problem, continued.

Practice Problems:

1. You're analyzing how quickly a video clip spreads online. You notice that the number of views increases rapidly and seems to grow by the same percentage each hour (not the same number of views, but the same percentage increase relative to the current total). Your analytics dashboards shows that the video has 100 views when first posted, and that it grows by 25% per hour while it trends. Develop a model. Then use your model to predict the number of views the video will have after 6 hours and how long it will take the video to reach 1,000 views.

2. You are less than a day away from getting your phone back after having to put it away 48 hours ago to go to the field over the summer. You promised your significant other know you'd call them the moment you got your phone back, but you need at least 10% battery to make the call. You realized that you forgot to turn off your phone and didn't put it in airplane mode. From past experience, you know your phone drops to 80% after sitting idle for 6 hours when not in airplane mode.

Assuming your phone's battery drains following an exponential decay pattern, where it loses the same proportion of its remaining charge each hour, determine the rate at which your phone battery drains and then use your model to determine if you will have enough battery to call your SO when you get your phone back.

3. You're on a weekend field training exercise and one of your battle buddies opens your Monster! Your battle quickly recaps the Monster, but you know it's only a matter of time before it goes flat. How long do you have to drink the Monster before it tastes flat now that it's been opened?

Context: The amount of dissolved carbon dioxide (CO_2) in a carbonated beverage decreases over time after it is opened. However, it never quite goes to zero. There is always a small amount of CO_2 that stays dissolved due to equilibrium with the surrounding air. Scientists have modeled this behavior using an exponential decay function:

$$C(t) = C_{air} + (C_0 - C_{air})e^{-kt}$$

where $C(t)$ is the amount of dissolved CO_2 in (g/L) at time t, C_0 is the initial amount of CO_2 when freshly opened, C_{air} is the equilibrium amount of CO_2 that remains in the beverage after it has gone flat, k is the decay rate per minute, and t is the time in minutes since the bottle was opened.

If $C_0 = 7.0$g/L, $C_{air} = 0.5$g/L, and the decay rate, k, is between 0.03 and 0.05 per minute, determine how long it takes the CO_2 level to drop to 1.5g/L, the point where most people say it tastes flat)?