

Admin Notes / Agenda

- Projects Due today.
- 5 lessons next week.
- See me for counseling, if you have not
- Complete survey
- Board Sheet

Key Terms:

1. **Exponential General Form** $y = ab^x + d$, where $a \rightarrow$ influences the initial value and vertical stretch, $b \rightarrow$ determines the growth or decay rate (general shape), and $d \rightarrow$ serves as a vertical shift.
2. **Exponential Growth / Decay Form** $y = a(1+r)^x + d$, where exponential functions are characterized by rapid growth when the base $b > 1$ or rapid decay when $0 < b < 1$. They also feature a horizontal asymptote, typically at $y = d$. Note: when $b < 0$, the function is undefined for most values of x .
3. **Functions** are parameterized mathematical models, we will use to predict outcomes
4. **explanatory variable**
5. **response variable**
6. **Families of Functions** linear, Exponential, and polynomial
7. **parameters** what we can change to affect our model selection
8. **Slope-intercept form of a line** $y = mx + b$ where m is the slope or rate of change and b is the y -intercept
9. **Point slope form of a line** $y - y_0 = m(x - x_0)$ where m is the slope or rate of change and (x_0, y_0) is a point on the line
10. **General form of a line** $Ax + By + C = 0$ where A , B , and C are constants.
11. In a **First Principle** approach a function is developed based on a physical or contextual relationship you understand about the problem
12. In an **Empirical** approach a solution type a function is developed based on data first. This maybe validated employing contextual information about the problem context or further data.
13. The **Sum of Square Error** (SSE) is a measure of error for your model to the data

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

14. The **Sum of Square Total** (SST) is a measure of un-normalized deviation of the data

$$SST = \sum_{i=1}^n (y_i - \bar{y})^2$$
15. The **slope** (m) of a line is a measure of rate of change between two points $m = \frac{y_2 - y_1}{x_2 - x_1}$
16. The **y-intercept** (b) of a line is a measure of rate of change between two points

$$b = y_1 - mx_1$$
17. The **The Coefficient of Determination** (R^2) is the percentage of the total observed variation in the response variable that is accounted for by changes in the explanatory variable $R^2 = 1 - \frac{SSE}{SST}$
18. **Interpolation:** Using a linear model to predict values *within the range* of the observed data.
19. **Extrapolation:** Using a linear model to predict values *outside the range* of the observed data, which is less reliable because the model may not hold beyond the data.

1 Negative Numbers Raised to Powers

What happens to a negative number raised to a power depends on the type of exponent.

1.1 Integer Exponents

- **Even exponent:** Multiplying a negative number an even number of times cancels the negatives, giving a positive result.

$$(-2)^2 = 4, \quad (-3)^4 = 81$$

- **Odd exponent:** Multiplying a negative number an odd number of times leaves one negative factor, giving a negative result.

$$(-2)^3 = -8, \quad (-5)^5 = -3125$$

1.2 Fractional Exponents

If the exponent is written as a fraction $\frac{p}{q}$ in lowest terms:

$$(-a)^{p/q} = \begin{cases} -(a^{p/q}), & q \text{ odd} \\ \text{not real}, & q \text{ even} \end{cases}$$

Example: $(-8)^{1/3} = -2$, $(-4)^{1/2}$ is not real.

3. Irrational Exponents

For irrational exponents (such as π or $\sqrt{2}$), the expression

$$(-a)^\alpha, \quad \alpha \notin \mathbb{Q}$$

is generally *not real* and is only defined in the complex number system.

Summary

$$(-a)^n = \begin{cases} +a^n, & n \in \mathbb{Z}, n \text{ even} \\ -a^n, & n \in \mathbb{Z}, n \text{ odd} \end{cases}$$

$$(-a)^{p/q} = \begin{cases} -(a^{p/q}), & q \text{ odd} \\ \text{not real}, & q \text{ even} \end{cases}$$

$(-a)^{\text{irrational}}$ is not real (complex-valued).

2 Model Family Discussion

2.1 Linear

Takes the slope-intercept form of $y = mx + b$.

Takes the point-slope form of $y - y_0 = m(x - x_0)$.

Takes the General form of a line of $Ax + By + C = 0$.

2.2 Exponential

Takes the form: $y = ab^x + d$

2.3 Polynomial

Takes the form:

$$y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

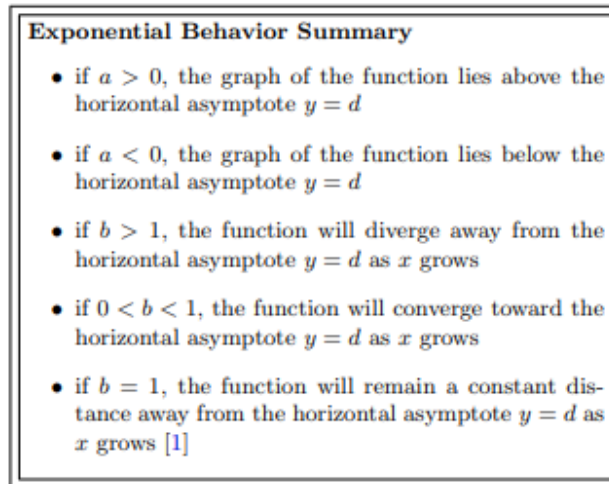


Figure 1: Exponential Behavior Explained

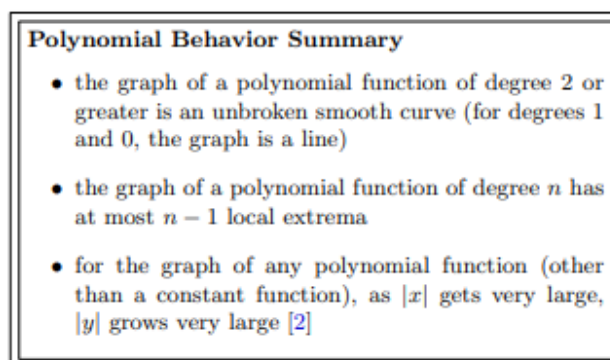


Figure 2: Polynomial Behavior Explained