Admin Notes / Agenda

- Quiz today
- See me for counseling, if you have not
- Complete survey
- Board Sheet

Key Terms:

1. A **polynomial** function takes the form:

$$y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where n is a nonnegative integer, and a_i is the i-th constant coefficient. The **degree** of the polynomial is n, which is the largest exponent of the independent variable x.

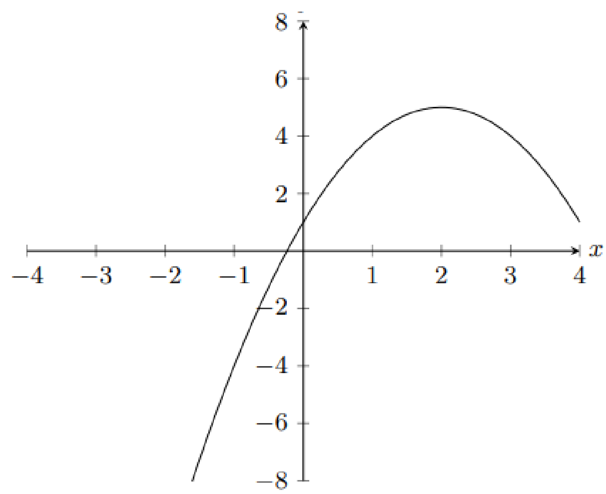

Figure 1: Graph of the quadratic function $y = -(x-2)^2 + 5$. This quadratic is in the form $y = a(x-b)^2 + d$. Notice that the sign of the leading coefficient a caused the graph to reflect over the line $y = d$, the constant b caused a translation in the positive x direction, and the constant d caused a translation in the positive y direction.

2. **Exponential General Form** $y = ab^x + d$, where $a \to$ influences the initial value and vertical stretch, $b \to$ determines the growth or decay rate (general shape), and $d \to$ serves as a vertical shift.

3. **Exponential Growth / Decay Form** $y = a(1+r)^x + d$, where exponential functions are characterized by rapid growth when the base $b > 1$ or rapid decay when $0 < b < 1$.They also feature a horizontal asymptote, typically at $y = d$. Note: when $b < 0$, the function is undefined for most values of x.

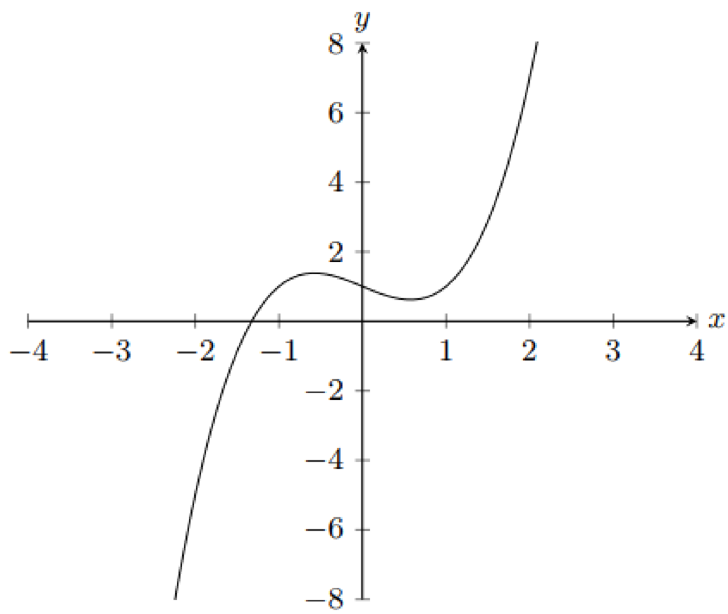

Figure 2: Graph of the cubic function $y = x^3 - x + 1$. Notice that as the degree of the polynomial increases, so does the number of possible turning points. As you progress in your mathematical career, you may recognize these as local extrema. A polynomial can have at most $n - 1$ local extrema, where n is the degree of the polynomial.

1 Model Family Discussion

1.1 Linear

Takes the slope-intercept form of $y = mx + b$.

Takes the point-slope form of $y - y_0 = m(x - x_0)$.

Takes the General form of a line of $Ax + By + C = 0$.

1.2 Exponential

Takes the form: $y = ab^x + d$

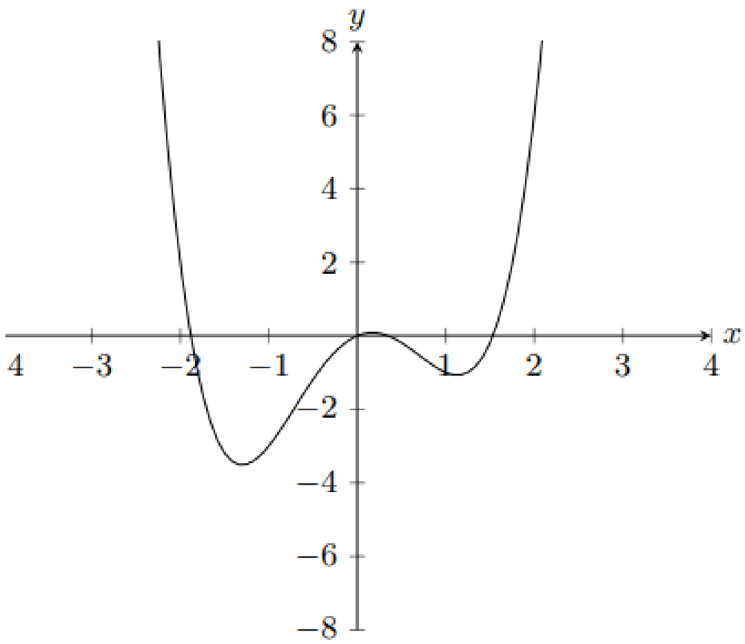

Figure 3: Graph of the degree 4 polynomial $y = x^4 - 3x^2 + x$. As we continue to increase the degree of our polynomials, notice that as $|x|$ increases, $|y|$ increases. In other words, as x increases, y either increases to infinity or decreases to negative infinity. Likewise, as x decreases, y either increases to infinity or decreases to negative infinity. This is the case for all polynomials, regardless of shape.

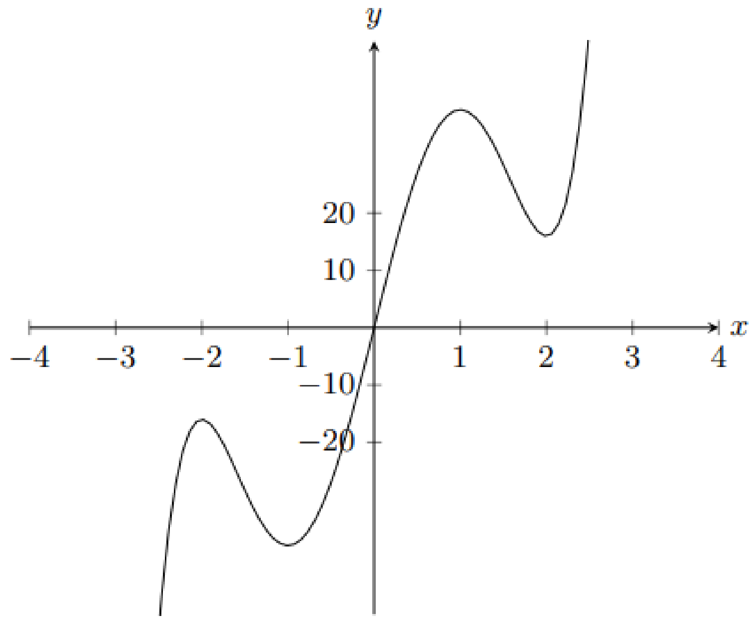

Figure 4: Graph of the degree 5 polynomial $y = 3x^5 - 25x^3 + 60x$. Notice that the general shape of the polynomial depends on if its degree, n, is even or odd. If n is even, then the polynomial is an even polynomial, and its graph is similar to a parabola in that y either increases or decreases as $|x|$ increases. If n is odd, then the polynomial is an odd polynomial, and its graph is similar to a cubic in that y both increases and decreases as $|x|$ increases.

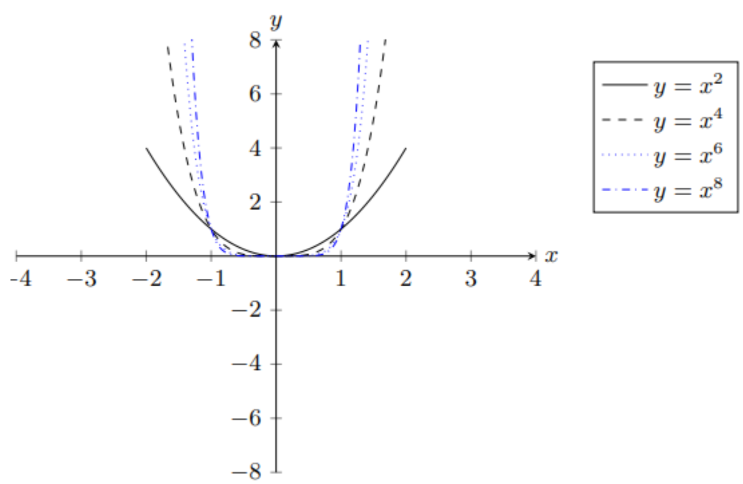

Figure 5: Even functions

Figure 5

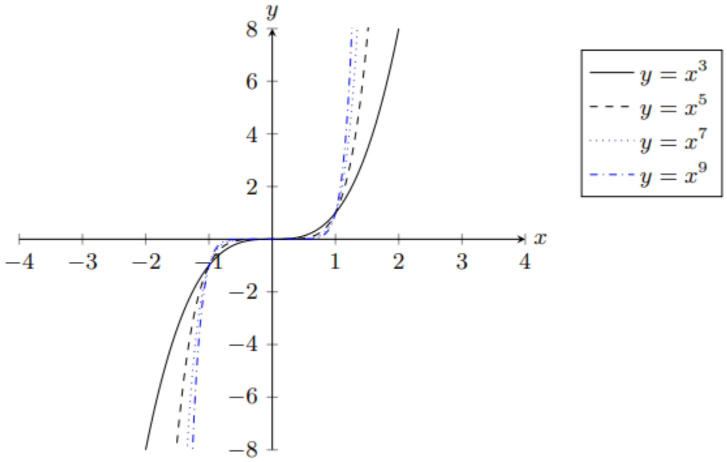

Figure 6: Odd functions

Figure 6

Polynomial Behavior Summary

- the graph of a polynomial function is an unbroken smooth curve
- the graph of a polynomial function of degree n has at most $n - 1$ local extrema
- for the graph of any polynomial function (other than a constant function), as $|x|$ gets very large, $|y|$ grows very large [1]

Figure 7

Exponential Behavior Summary

- if $a > 0$, the graph of the function lies above the horizontal asymptote $y = d$
- if $a < 0$, the graph of the function lies below the horizontal asymptote $y = d$
- if $b > 1$, the function will diverge away from the horizontal asymptote $y = d$ as x grows
- if $0 < b < 1$, the function will converge toward the horizontal asymptote $y = d$ as x grows
- if $b = 1$, the function will remain a constant distance away from the horizontal asymptote $y = d$ as x grows [1]

Figure 8: Exponential Behavior Explained