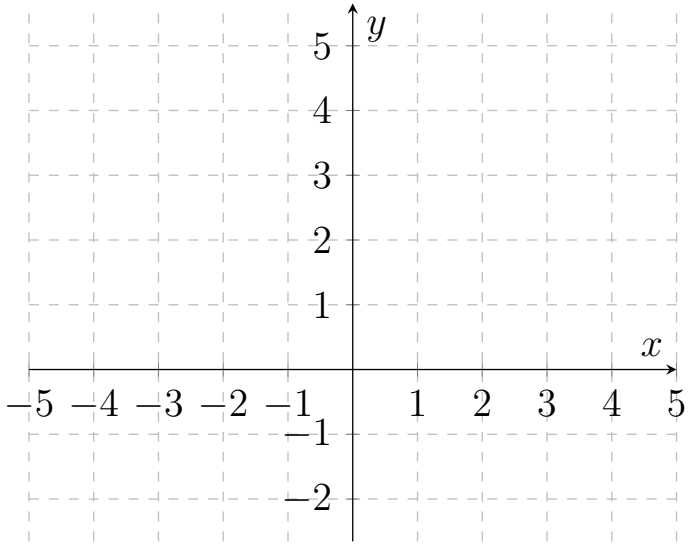
MA103: Mathematical Modeling

- Understand how vectors can be used both to describe things in physical space and also to represent mathematical arrays that are not necessarily related to physical space
- Be familiar with vector notations and terminology
- Perform vector addition/subtraction and multiplication of a vector by a scalar
- Compute and interpret dot products of vectors
- Understand when vector operations are possible based on dimensions
- Use vectors to solve modeling problems

Vectors:

On the grid below, draw the position vector $\vec{a}$ and $\vec{b}$ for $A(-3,1)$ and $B(2,3)$, respectively.



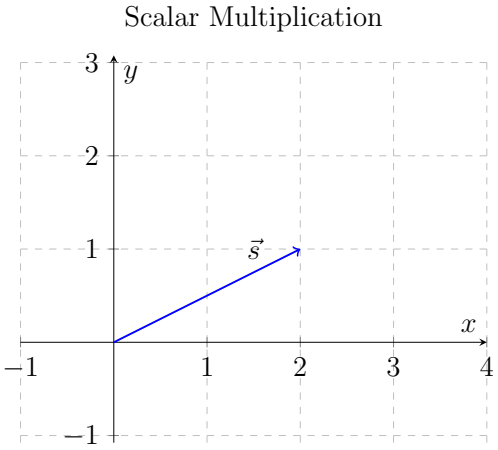
Now draw the displacement vector $\overrightarrow{AB}$ from A to B.

Draw the position vector $\vec{d} = \langle 5, 2 \rangle$. Is this the same vector as displacement vector $\overrightarrow{AB}$?

Vector Operations:

Scalar multiplication

Given scalar $c = 2$ and vector $\vec{s} = \langle 2, 1 \rangle$, calculate $c\vec{s}$ and then graph the resulting vector.



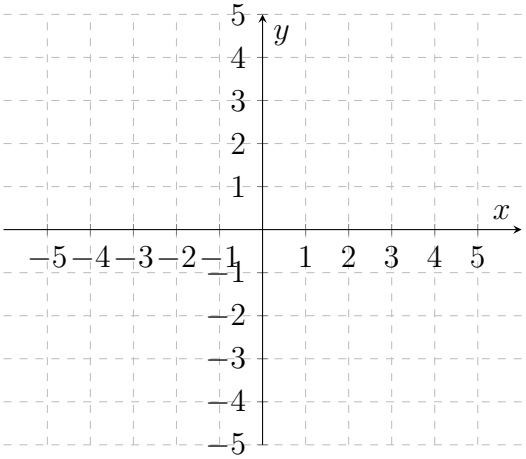
Vector addition and subtraction

Given vector $\vec{u} = \langle 4, 2 \rangle$ and $\vec{v} = \langle -1, 3 \rangle$. Solve each algebraically and graphically using the grids below.

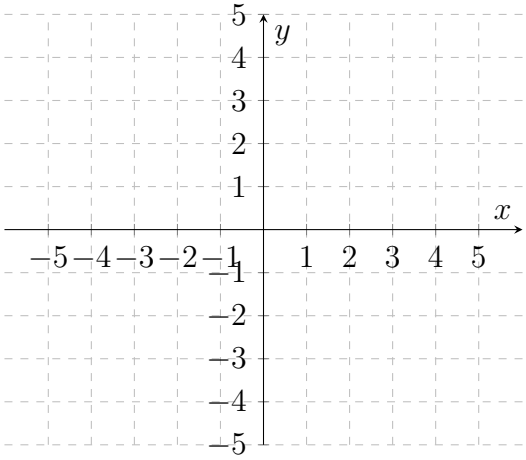
1. $\vec{u} + \vec{v}$

2. $\vec{v} - \vec{u}$

1. Vector Addition



2. Vector Subtraction



Note: To solve graphically use the _____ method.

What does $-\vec{u}$ represent?

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1. What is the **magnitude** of a vector?

2. Given a vector $\vec{w}$ with a length of 13 and an angle of 22.62°measured counter clockwise from the positive x-axis, what is $\vec{w}$ written in component notation?

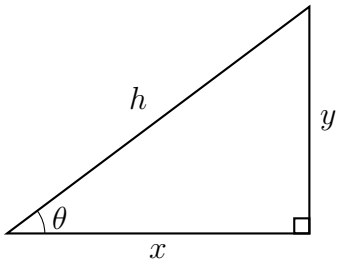
3. What is a **unit vector**?

What is the unit vector $(\hat{s})$, for $\vec{s} = \langle 3, 4 \rangle$?

4. What are the **standard basis vectors**?

Write the vector $\vec{r} = \langle 4, 2, -7 \rangle$ as a linear combination of the standard basis vectors.

Trig Review:



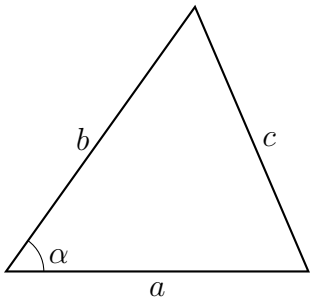
figureRight

Triangle

$$\sin\theta = \frac{y}{h}$$

$$\cos\theta = \frac{x}{h}$$

$$\tan\theta = \frac{y}{x}$$



Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\alpha$$

Dot Product:

1. Calculate the following, given $\vec{u} = \langle -2, 4, 0 \rangle$, $\vec{v} = \langle \frac{1}{2}, 5, -2 \rangle$, $\vec{w} = \langle 1, -1, 2 \rangle$, $\vec{x} = \langle -4, 6 \rangle$. If you cannot compute one explain why.

Problem 1
Solve the following problems:

a) Two forces act on a point: $\vec{F_1} = \langle 4, 3 \rangle$ and $\vec{F_2} = \langle -2, 6 \rangle$. Find the resultant force vector $\vec{F_r} = \vec{F_1} + \vec{F_2}$.

b) An object moves from point $A(2, 5)$ to point $B(8, 9)$. What is the displacement vector $\overrightarrow{AB}$?

c) A plane travels with a velocity of $\vec{v_1} = \langle 200, 100, 50 \rangle$ km/h relative to the air, and the wind adds a velocity of $\vec{v_2} = \langle 20, -30, 10 \rangle$ km/h. What is the plane's total velocity $\vec{v}_{\text{total}}$?

d) A force vector $\vec{F} = \langle 3, -4 \rangle$ is scaled by a factor of 5. Find the new force vector.

e) A drone moves with velocity $\vec{v} = \langle 2, 3, 4 \rangle$ m/s. If its speed doubles, what is the new velocity vector?

f) Find the magnitude of the velocity vector $\vec{v} = \langle 7, -24 \rangle$.

g) A submarine travels with velocity $\vec{v} = \langle 10, 20, 30 \rangle$ m/s. What is its speed (magnitude of $\vec{v}$)?

h) A displacement vector is given as $\vec{d} = \langle -5, 12 \rangle$. Find the angle θ (in degrees) that the vector makes with the positive x-axis.

i) Two velocity vectors are combined: $\vec{v_1} = \langle 3, 4 \rangle$ and $\vec{v_2} = \langle 1, -2 \rangle$.

- Find the resultant vector $\vec{v_r} = \vec{v_1} + \vec{v_2}$.
- Determine the magnitude and direction (angle with the positive x-axis) of $\vec{v_r}$.

j) A car travels 50 km at an angle of 30° north of east.

- Write the vector representing the car's displacement in component form.