MA103: Mathematical Modeling
Vectors II **05 December 2024**

1. Compute and interpret dot products of vectors.
2. Understand when vector operations are possible based on dimensions.
3. Use vectors to solve modeling problems.

Warm Up:
Given $\vec{u} = \langle 2, 4 \rangle$ and $\vec{v} = \langle -1, 1 \rangle$ determine the magnitude and direction (in degrees counterclockwise from the x-axis) of both vectors.

Dot Product Problems with Solutions:

1. $\vec{u} \cdot \vec{v}$
   $-2 * 0.5 + 4 * 5 + 0 * (-2) = -1 + 20 + 0 = 19$

2. $\vec{v} \cdot \vec{u}$
   Same as above: 19

3. $\vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
   $= 19 + (-2 * 1 + 4 * (-1) + 0 * 2) = 19 - 2 - 4 + 0 = 13$

4. $\vec{u} \cdot (\vec{v} + \vec{w})$
   $\vec{v} + \vec{w} = \langle 1.5, 4, 0 \rangle$; $\vec{u} \cdot (\vec{v} + \vec{w}) = -2 * 1.5 + 4 * 4 + 0 * 0 = -3 + 16 = 13$

5. $\vec{w} \cdot \vec{x}$
   Cannot compute: dimensions mismatch

6. $\vec{u} \cdot \vec{u}$
   $(-2)^2 + 4^2 + 0^2 = 4 + 16 + 0 = 20$

7. $|\vec{u}|^2$
   20

8. Prove $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$
   By definition from geometric interpretation.

9. Given $|\vec{a}| = 2\sqrt{6}$, $|\vec{b}| = 2\sqrt{3}$, angle $= 45$
   $\vec{a} \cdot \vec{b} = 2\sqrt{6} \cdot 2\sqrt{3} \cdot \cos(45°) = 4\sqrt{18} \cdot \frac{\sqrt{2}}{2} = 2\sqrt{36} = 12$

10. $\vec{r} = \langle -2, 4 \rangle$, $\vec{s} = \langle 1, 3 \rangle$
    $\vec{r} \cdot \vec{s} = -2 * 1 + 4 * 3 = -2 + 12 = 10$; $|\vec{r}| = \sqrt{20}$, $|\vec{s}| = \sqrt{10}$; $\theta = \cos^{-1}\left(\frac{10}{\sqrt{200}}\right)$

11. Which vectors are orthogonal?
    $\vec{u} \cdot \vec{c} = 0$ orthogonal; $\vec{u} \cdot \vec{d} = 26$ not orthogonal; $\vec{c} \cdot \vec{d} = 0$ orthogonal. So (u,c) and (c,d) are perpendicular.

12. Work done by 30lb force at 60over 10ft
    $W = 30 \cdot 10 \cdot \cos(60°) = 150$ ft-lb

13. AAA Party Supply Store sales and profit
    Sales = 16267.5; Cost = 1883.8; Profit = 14383.7