

Admin Notes / Agenda

- PSL work continued
- dot product warm up
- Lecture - larger matrix
- Board sheets
- Lecture - transpose and inverse

Warm Up: From Dot Product to Matrix Multiplication

Compute the following products. Show your work by multiplying corresponding entries and adding.

Part A: Dot Products (Row \times Column)

1. $[3 \ 2] \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

2. $[-2 \ 5 \ 1] \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

Part B: Multiple Dot Products in One Problem

Use the same dot-product idea to compute each entry of the resulting matrix.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 0 & 5 \end{bmatrix}$$

3. Compute AB by finding:
- Row 1 of A dotted with Column 1 of B
 - Row 1 of A dotted with Column 2 of B
 - Row 2 of A dotted with Column 1 of B
 - Row 2 of A dotted with Column 2 of B

Part C: Non-Square Matrix Multiplication

$$C = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}$$

4. Compute CD . (Hint: There will be 4 dot products total.)

Summary of Key Terms and Operations

- **Matrix:** A matrix is a rectangular array of numbers arranged in rows and columns. A matrix with m rows and n columns is called an $m \times n$ matrix.
- **Matrix Notation:** A matrix is labeled with a capital letter (e.g., A , B), and its elements with subscripts. For example:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

where a_{ij} is the entry in the i th row and j th column.

- **Rows and Columns:**
 - A *row* is a horizontal selection of elements in a matrix.
 - A *column* is a vertical selection of elements in a matrix.
- **Vectors as Matrices:** A vector in component form can be written as a matrix:
 - A *row vector* is a $1 \times n$ matrix, e.g. $[2 \quad -1 \quad 5]$
 - A *column vector* is an $n \times 1$ matrix, e.g. $\begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$

Both forms represent the same vector, just written differently for different operations.

- **Matrix Addition:** Two matrices must have the same dimensions. Add corresponding entries:

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

- **Matrix Subtraction:** Subtract corresponding entries (same-sized matrices only):

$$A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{bmatrix}$$

- **Scalar Multiplication:** Multiply each entry of a matrix by a real number c :

$$cA = \begin{bmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{bmatrix}$$

Matrix Multiplication

Matrix multiplication combines rows of the first matrix with columns of the second using the **dot product**.

Step 1: Start with the dot product.

$$[2 \quad 3] \begin{bmatrix} 4 \\ 1 \end{bmatrix} = 2(4) + 3(1) = 11$$

A 1×2 row vector times a 2×1 column vector gives a single number (a scalar).

Step 2: Extend to a 2×2 matrix multiplication.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1)(2) + (2)(1) & (1)(0) + (2)(5) \\ (3)(2) + (4)(1) & (3)(0) + (4)(5) \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 10 & 20 \end{bmatrix}$$

Each entry is found by taking a row of A dotted with a column of B .

Step 3: A non-square example (2×3 times 3×2).

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1)(3) + (0)(2) + (2)(1) & (1)(1) + (0)(1) + (2)(0) \\ (-1)(3) + (3)(2) + (1)(1) & (-1)(1) + (3)(1) + (1)(0) \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$$

The result is a 2×2 matrix.

Matrix Transpose

The **transpose** of a matrix, written A^T , is formed by swapping rows and columns.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Explanation: - The first row of A becomes the first column of A^T . - The second row of A becomes the second column of A^T .

A is 2×3 , so A^T is 3×2 .

Identity Matrix

The **identity matrix**, denoted I , acts like the number 1 for matrix multiplication:

$$AI = IA = A$$

For 2×2 matrices:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Example:

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

Similarly, for 3×3 :

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Inverse of a Matrix

The **inverse** of a matrix A , written A^{-1} , is defined (for square matrices only) so that:

$$AA^{-1} = A^{-1}A = I$$

Not all matrices have inverses — only those with a nonzero determinant.

For a 2×2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example:

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{(2)(3) - (1)(5)} \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \times \frac{1}{1}$$

Verification:

$$AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

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