Admin Notes / Agenda

- Warm Up
- Lesson Review
- Board Sheet

Hiker's Route Planning Problem

A hiker studies a **topological map** showing elevation contours that rise toward a single peak (see Figure 1). The hiker's goal is to reach the highest possible elevation *without leaving the permitted region* outlined by straight boundary lines on the map.

The allowed region is defined by the following linear constraints:

$$\begin{aligned} 2x + y &\leq 120 && \text{(northern boundary)} \\ x + 2y &\leq 110 && \text{(eastern boundary)} \\ x \geq 0, \quad y &\geq 0 && \text{(southern and western limits)} \end{aligned}$$

The hiker must stay within this polygonal region while seeking the point of highest elevation on the map.

1. Map the constraints
2. Solve for the intersection point using the Inverse Matrix Method
3. Compare this to what you can visually identify as the highest point on the map

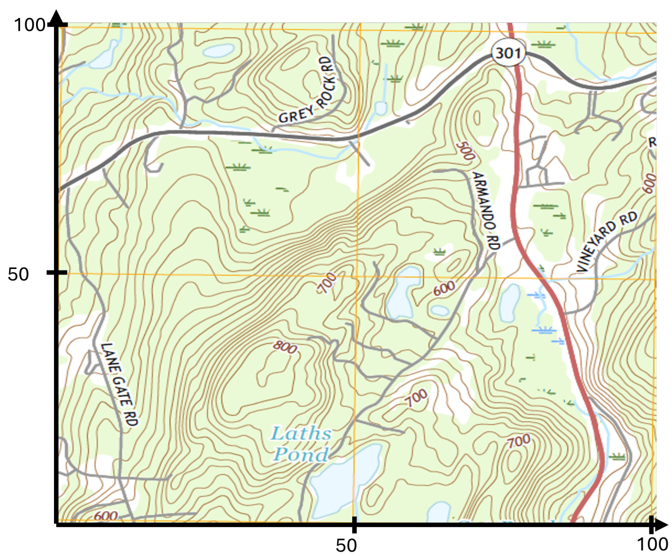

Figure 1: Topological map showing contour lines and the feasible region defined by the linear constraints.

1 Linear Programming Assumptions

Linear programming models rely on several fundamental assumptions that ensure the relationships between variables remain linear and mathematically tractable:

1. **Additivity:** The total effect of all decision variables is the sum of their individual contributions. There are no interactions or combined effects among variables.

   $$Z = 5x_1 + 3x_2$$

   Here, the total profit Z is simply the sum of the profits from x_1 and x_2; there is no term like x_1x_2, which would violate additivity.

2. **Proportionality:** Each variable's contribution to the objective function and constraints is directly proportional to its value. Doubling a variable doubles its impact on cost, profit, or resource use.

   $$\text{If } x_1 = 2 \text{ and } a_1x_1 = 10, \text{ then doubling } x_1 \text{ to 4 makes } a_1x_1 = 20.$$

   This linear relationship holds because a_1x_1 scales directly with x_1.

3. **Continuity:** Decision variables can take on any fractional (continuous) value within the feasible region. This means the model assumes quantities are divisible rather than restricted to whole numbers.

   $$x_1 = 2.5 \text{ units is allowed, as long as it satisfies all constraints.}$$

   For example, $2x_1 + 3x_2 \leq 20$ remains valid even if $x_1 = 2.5$ and $x_2 = 4.3$.