

MA103: Mathematical Modeling & Intro to Calculus

Constraints and Feasible Regions 1

Lesson Objectives: Cadets will

1. Understand what it means for a solution to be feasible in the context of a linear programming problem
 2. Identify whether a potential solution is feasible or not
 3. Represent constraints algebraically
 4. Visualize constraints for a linear programming problem as a feasible region (by hand)
 5. Explore feasible regions and compare solutions within the feasible region
 6. Explain the four model-driven assumptions that are necessary to use a linear programming model, and justify each for a real-world modeling scenario
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Recall our introductory problem: Your Engineer Platoon has 2 different MOS types: 12B and 12N. You can build Anti-Tank Ditches (ATD) and 11-Row wire obstacles. The ATD requires 3 x 12N Soldiers and 1 x 12B Soldier. The 11-Row requires 1 x 12N and 2 x 12Bs. You only have 9 x 12Ns and 8 x 12Bs. Each ATD will span 300 meters and the 11 Row will block 200 meters. How many of each type should your Platoon build to maximize the amount of perimeter (in meters) you can protect?

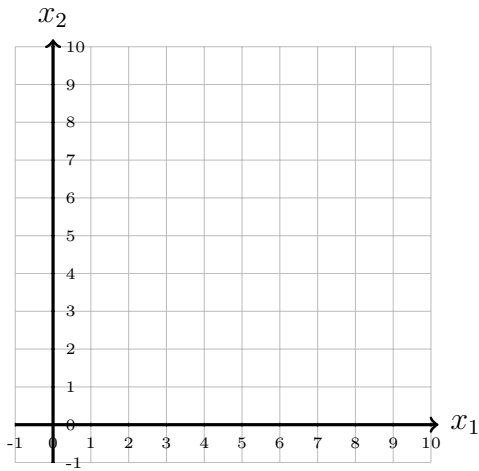
$$\begin{aligned} \max_{x_1, x_2} \quad & P = 300x_1 + 200x_2 \\ \text{s.t.} \quad & 3x_1 + x_2 \leq 9 \\ & x_1 + 2x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned} \tag{1}$$

Write the objective function from our formulation:

Write the constraints:

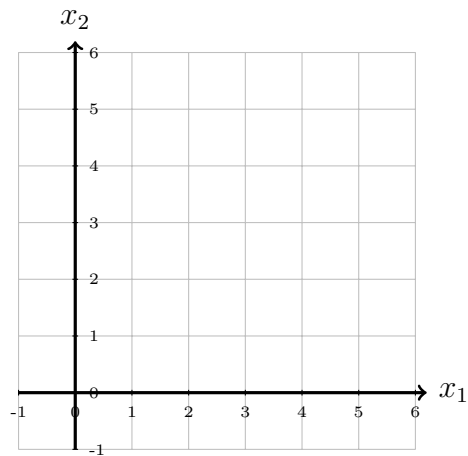
What does it mean for a solution to this problem to be **feasible**?

How can we visualize the possible feasible solutions to this problem?



NTC Equipment: You are preparing to ship equipment for the advance echelon of a unit deployment. You must ship at least 4 tons of equipment by some combination of line haul or rail head. You are allotted 3 tons to ship via rail at a cost of \$2 per ton. You are allotted 4 tons of shipping via truck at \$6 per ton (all dollar amounts are in hundreds of dollars). All of your equipment must arrive in at most 12 days. Line haul is typically much faster and takes an average of 2 days per ton while rail takes an average of 3 days per ton. Formulate a linear program that minimizes the cost of shipping equipment to NTC.

Formulation:



Farming Problem: Stoneridge Farm recently acquired two additional 50 acre fields of arable land. The farm management team is working to determine what mix of crops to plant. At first they want to try alfalfa, but decide that it is too costly to install the additional irrigation infrastructure required to grow alfalfa. The team settles on corn and wheat. Corn yields a profit of \$1,000 per acre and wheat yields \$875 per acre. The farm has a total budget of \$15,000 and \$2,800 for fertilizer and seed costs respectively. The farm has 300 total man-hours available. Corn requires \$200 and \$30 per acre in fertilizer and seed respectively, and requires 3 man-hours per acre. Wheat requires \$160 and \$40 in fertilizer and seed costs, and requires 4 man-hours per acre. How many acres of corn and wheat should the management team plant to maximize profit?

Algebraic Formulation:

HMMWVs and Ammunition:

A company produces HMMWV engines and ammunition using three plants: plant 1 produces steel engine parts, plant 2 produces brass casings, plant 3 assembles the HMMWVs and ammunition. Plant 1 will be available 3 hours per week, Plant 2 will be available 15 hours per week, and Plant 3 will be available 15 hours per week. Each HMMWV requires 1 hour of production time in Plant 1 and 3 hours in Plant 3. Each pallet of ammunition requires 2 hours of production time in Plant 2 and 2 hours in Plant 3. The profit per unit will be \$3000 per HMMWV and \$1000 per pallet of ammunition. What combination of production rates (the number of units produced per week) for the two products would maximize total profit?

Algebraic Formulation: