Admin Notes / Agenda

- Warm Up - hiking problem
- Lesson Review - CDT Yoo
- Lecture
- Board Sheet

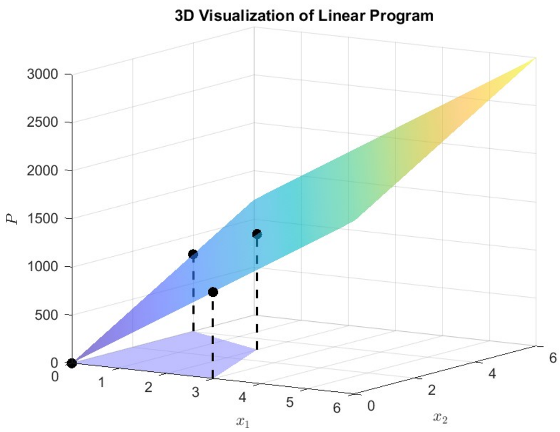
So far we've focused on constraint formulation, let's shift focus to the objective and solving our linear programs. We'll start with a familiar problem:

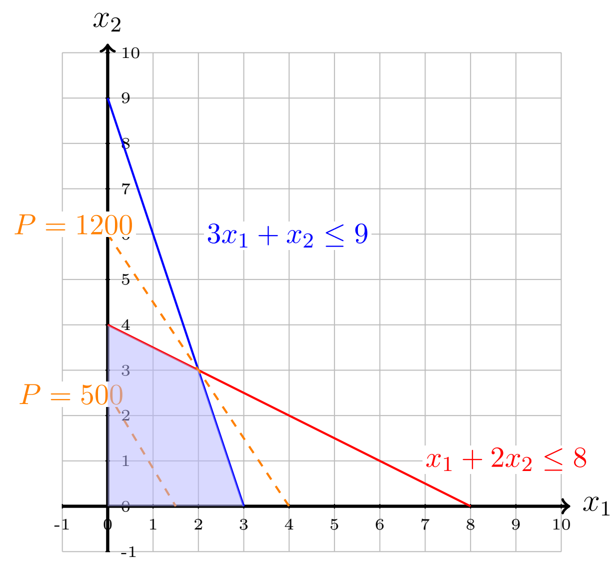
Your Engineer Platoon has 2 different MOS types: 12B and 12N. 12B Soldiers are combat engineers, 12Ns are heavy equipment operators. You're preparing a defense against an enemy Armored Company. There are two types of obstacles you can build: an Anti-Tank Ditch (ATD) and an 11-Row wire obstacle. The ATD requires 3 x 12N Soldiers and 1 x 12B Soldier. The 11-Row requires 1 x 12N and 2 x 12Bs. You only have 9 x 12Ns and 8 x 12Bs. Each ATD will span 300 meters and the 11 Row will block 200 meters. How many of each type should your Platoon build to maximize the amount of perimeter (in meters) you can protect?

$$
\begin{aligned}
\max_{x_1,x_2} \quad & P = 300x_1 + 200x_2 \\
\text{s.t.} \quad & 3x_1 + x_2 \leq 9 \\
& x_1 + 2x_2 \leq 8 \\
& x_1, x_2 \geq 0
\end{aligned} \tag{1}
$$

We know what the feasible region looks like for this problem, but how do we know what the optimal solution is?

Two ways to visualize the objective function:





What do you notice about the location of the optimal solution? Do you think this will always be the case?

1 Linear Programming Assumptions

Linear programming models rely on several fundamental assumptions that ensure the relationships between variables remain linear and mathematically tractable:

1. **Additivity:** The total effect of all decision variables is the sum of their individual contributions. There are no interactions or combined effects among variables.

$$Z = 5x_1 + 3x_2$$

   Here, the total profit Z is simply the sum of the profits from x_1 and x_2; there is no term like x_1x_2, which would violate additivity.

2. **Proportionality:** Each variable's contribution to the objective function and constraints is directly proportional to its value. Doubling a variable doubles its impact on cost, profit, or resource use.

$$\text{If } x_1 = 2 \text{ and } a_1x_1 = 10, \text{ then doubling } x_1 \text{ to 4 makes } a_1x_1 = 20.$$

   This linear relationship holds because a_1x_1 scales directly with x_1.

3. **Continuity:** Decision variables can take on any fractional (continuous) value within the feasible region. This means the model assumes quantities are divisible rather than restricted to whole numbers.

$$x_1 = 2.5 \text{ units is allowed, as long as it satisfies all constraints.}$$

   For example, $2x_1 + 3x_2 \leq 20$ remains valid even if $x_1 = 2.5$ and $x_2 = 4.3$.