

MA103: Mathematical Modeling & Intro to Calculus

Objective Functions and Solving 1

Lesson Objectives: Cadets will

1. Describe the difference between a feasible solution and an optimal solution
 2. Identify the objective function for a linear program
 3. Represent the objective function algebraically
 4. Use the graphical method to solve a modeling problem using linear programming
 5. Set up and solve a modeling problem with linear programming using Excel Solver
 6. Interpret an optimal solution in the context of a modeling problem and articulate the optimal solution to a stakeholder
 7. Understand how different objective functions can lead to different optimal solutions
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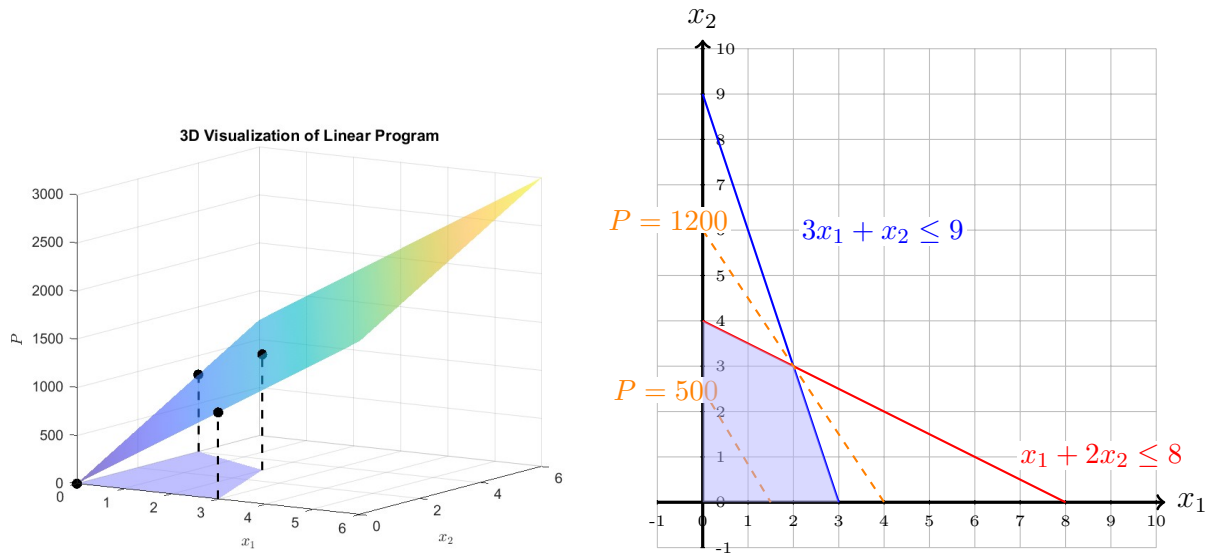
So far we've focused on constraint formulation, let's shift focus to the objective and solving our linear programs. We'll start with a familiar problem:

Your Engineer Platoon has 2 different MOS types: 12B and 12N. 12B Soldiers are combat engineers, 12Ns are heavy equipment operators. You're preparing a defense against an enemy Armored Company. There are two types of obstacles you can build: an Anti-Tank Ditch (ATD) and an 11-Row wire obstacle. The ATD requires 3 x 12N Soldiers and 1 x 12B Soldier. The 11-Row requires 1 x 12N and 2 x 12Bs. You only have 9 x 12Ns and 8 x 12Bs. Each ATD will span 300 meters and the 11 Row will block 200 meters. How many of each type should your Platoon build to maximize the amount of perimeter (in meters) you can protect?

$$\begin{aligned} \max_{x_1, x_2} \quad & P = 300x_1 + 200x_2 \\ \text{s.t.} \quad & 3x_1 + x_2 \leq 9 \\ & x_1 + 2x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned} \tag{1}$$

We know what the feasible region looks like for this problem, but how do we know what the optimal solution is?

Two ways to visualize the objective function:



What do you notice about the location of the optimal solution? Do you think this will always be the case?

NTC Equipment: You are preparing to ship equipment for the advance echelon of a unit deployment. You must ship at least 4 tons of equipment by some combination of line haul or rail head. You are allotted 3 tons to ship via rail at a cost of \$2 per ton. You are allotted 4 tons of shipping via truck at \$6 per ton (all dollar amounts are in hundreds of dollars). All of your equipment must arrive in at most 12 days. Line haul is typically much faster and takes an average of 2 days per ton while rail takes an average of 3 days per ton. **Solve for and interpret the optimal solution.**

$$\begin{aligned}
 \max_{x_1, x_2} \quad & C = 2x_1 + 6x_2 \\
 \text{s.t.} \quad & 3x_1 + 2x_2 \leq 12 \\
 & x_1 + x_2 \geq 4 \\
 & x_1 \leq 3 \\
 & x_2 \leq 4 \\
 & x_1, x_2 \geq 0
 \end{aligned} \tag{2}$$