

## Admin Notes / Agenda

- Download prior quizzes and exams NLT 30 NOV
- Warm Up
- PSL

# 1 Key Definitions for What If

Linear programming models rely on several fundamental assumptions that ensure the relationships between variables remain linear and mathematically tractable:

1. **binding:** A constraint is binding if the left-hand side and the right-hand side of the constraint are equal when the optimal values of the variables are substituted into the constraint.
2. **non-binding:** A constraint is non-binding if the left-hand side and the right-hand side of the constraint are unequal when the optimal values of the decision variables are substituted into the constraint.
3. **Shadow Price:** A shadow price is how much the objective function value is increased if the right-hand side value of a binding constraint is increased by 1, as long as it remains a binding constraint.
4. **What if: Interpret slopes of the objective:** For the objective function  $Z = c_1x_1 + c_2x_2$ , each pair  $(c_1, c_2)$  defines a contour line whose slope is  $-\frac{c_1}{c_2}$ . Sensitivity analysis in two dimensions therefore reduces to determining the range of slopes for which the contour line still supports the feasible region at the current optimal vertex. The allowable range is the interval of slopes for which no adjacent vertex yields a better objective value, ensuring that the supporting line touches the feasible region only at the optimal point.

## 2 Some Review:

1. **Solving for an unknown component of a velocity vector:** A particle moves with velocity

$$\vec{v} = \langle 3, v_2 \rangle$$

and the speed is

$$\|\vec{v}\| = 10.$$

Find the unknown component  $v_2$ .

2. **Finding the angle between two velocity vectors:** Two objects move with velocities

$$\vec{u} = \langle 4, -2 \rangle, \quad \vec{w} = \langle 1, 5 \rangle.$$

Calculate the angle between  $\vec{u}$  and  $\vec{w}$ .

3. **Solving a 2D system using a matrix inverse:** Consider the system

$$\begin{cases} 2x + 3y = 7, \\ -4x + y = -5. \end{cases}$$

- (a) Write this system in the form  $A\vec{x} = \vec{b}$ .  
 (b) Compute  $A^{-1}$  and use it to solve  $\vec{x} = A^{-1}\vec{b}$ .  
 (c) Describe the solution as the intersection point of the two lines.
4. **Finding a missing vector component from magnitude:** A force vector is given by

$$\vec{F} = \langle F_1, -6 \rangle,$$

and it is known that the magnitude of the force is

$$\|\vec{F}\| = 10.$$

Determine the missing component  $F_1$ .

5. **Compute an angle between two acceleration vectors:** Given

$$\vec{a}_1 = \langle -3, 4 \rangle, \quad \vec{a}_2 = \langle 2, 1 \rangle,$$

find the angle between the two acceleration vectors.

6. **Matrix multiplication (2x2 times 2x1):** Compute the product

$$\begin{pmatrix} 3 & -1 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Write the resulting vector in simplified form.

7. **List the four assumptions of linear programming:** List and briefly describe the four main assumptions of linear programming: additivity, proportionality, divisibility, and certainty.

### 3 Feasible Region Analysis and Objective Function Exploration

1. **Draw the feasible region and identify all intersection points and the optimal point.** Using the given system of linear constraints, sketch the feasible region in the  $x_1$ - $x_2$  plane. Clearly mark every corner point (vertex) formed by intersections of constraint boundaries, and determine which vertex is optimal for the original objective function.
2. **Explain why we may limit our search for optimal points to the vertices of the feasible region.** Provide a brief justification, using the principles we have discussed, for why linear programs achieve their optimum at a vertex of the feasible region rather than in the interior.
3. **Identify a new objective function and justify your choice.** Propose an alternative objective function and explain the motivation behind selecting new coefficients. Describe how this new objective direction changes the orientation of objective contour lines and how it may alter the optimal solution.
4. **Determine the allowable range for the cost of local fishing licenses that preserves the original optimal solution, and interpret the result.** Returning to the original objective function, treat the coefficient associated with local fishing licenses as a variable parameter. Determine the interval of values for this coefficient such that the current optimal vertex remains optimal. Interpret the meaning of this allowable range in practical terms—what does it tell us about price flexibility before the optimal decision changes?