MA103: Mathematical Modeling & Intro to Calculus
What-if Analysis 1

Lesson Objectives: Cadets will

1. Understand how changes to non-binding constraints impact the feasible region and the optimal solution
2. Understand how changes to binding constraints impact the feasible region and the optimal solution
3. Understand how changes in the objective function can change the optimal solution
4. Explore what-if analysis graphically
5. Interpret the results of a what-if analysis in the context of a modeling problem

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A Certain Bread Company

A company produces 2 types of lemon flavored drinks: Regular and Charged Lemonade. The number of Charged Lemonade drinks is limited by machine mixing capacity and is limited to 10 per hour. Your pantry also has limited space to store lemons for processing. The regular Lemonade requires 2 lemons per large drink and the Charged one requires 5 lemons. You can only fit 60 lemons at a time in the pantry. The lemonades are also served in special cups and you only have 18 cups total available per hour. Finally, you are limited by the amount of ice your ice machine can make. The regular Lemonade usually has 3 scoops of ice per large drink, while the Charged Lemonade is full of caffeine and chemicals which leaves less room for ice. The Charged Lemonade only requires 1 scoop of ice per drink. Your ice machine only makes enough ice to fill 44 scoops total every hour. The Regular Lemonade generates a profit of \$2 and the Charged Lemonade only generates \$1 per drink. How many of each type of Lemonade should the bread company make every hour to maximize their profit?

$$\begin{aligned} \max_{x_1,x_2} \quad & P = 2x_1 + 1x_2 \\ \text{s.t.} \quad & x_2 \leq 10 \quad Machine\,Capacity \\ & 2x_1 + 5x_2 \leq 60 \quad Lemons \\ & x_1 + x_2 \leq 18 \quad Cups \\ & 3x_1 + x_2 \leq 44 \quad Ice \\ & x_1, x_2 \geq 0 \quad Non - Negativity \end{aligned} \tag{1}$$

The feasible region for this problem can be found here: https://www.desmos.com/calculator/ecfa70ff27

1. Which constraints are **binding**?

2. Which constraints are **non-binding**?

3. What if machine capacity decreases? How much can machine capacity decrease before it becomes a binding constraint? For what range of machine capacity is the optimal solution still optimal?

4. If we change the available cups from 18 to 19, will the optimal solution change? By how much does the objective function change?

5. What is the *shadow price* of cups?

6. If the number of lemons required to make regular lemonade increases to 3, will the optimal solution change?For what range of lemons is the optimal solution still optimal?

7. How much can the number of available lemons change before the constraint becomes binding?