MA103: Mathematical Modeling & Intro to Calculus
What-if Analysis 3

Lesson Objectives: Cadets will

1. Understand how changes to non-binding constraints impact the feasible region and the optimal solution
2. Understand how changes to binding constraints impact the feasible region and the optimal solution
3. Understand how changes in the objective function can change the optimal solution
4. Explore what-if analysis graphically
5. Interpret the results of a what-if analysis in the context of a modeling problem

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Recall: A Certain Bread Company

A company produces 2 types of lemon flavored drinks: Regular and Charged Lemonade. The number of Charged Lemonade drinks is limited by machine mixing capacity and is limited to 10 per hour. Your pantry also has limited space to store lemons for processing. The regular Lemonade requires 2 lemons per large drink and the Charged one requires 5 lemons. You can only fit 60 lemons at a time in the pantry. The lemonades are also served in special cups and you only have 18 cups total available per hour. Finally, you are limited by the amount of ice your ice machine can make. The regular Lemonade usually has 3 scoops of ice per large drink, while the Charged Lemonade is full of caffeine and chemicals which leaves less room for ice. The Charged Lemonade only requires 1 scoop of ice per drink. Your ice machine only makes enough ice to fill 44 scoops total every hour. The Regular Lemonade generates a profit of \$2 and the Charged Lemonade only generates \$1 per drink. How many of each type of Lemonade should the bread company make every hour to maximize their profit?

$$\begin{aligned} \max_{x_1,x_2} \quad & P = 2x_1 + 1x_2 \\ \text{s.t.} \quad & x_2 \leq 10 \quad Machine\,Capacity \\ & 2x_1 + 5x_2 \leq 60 \quad Lemons \\ & x_1 + x_2 \leq 18 \quad Cups \\ & 3x_1 + x_2 \leq 44 \quad Ice \\ & x_1, x_2 \geq 0 \quad Non - Negativity \end{aligned} \tag{1}$$

Today we will ask some more interesting questions about our lemonade problem!

Find Optimal: Given the corner points for this problem are: (0,0) (13,5) (10,8) (5,10) (0,10) and (14.66,0). What is the optimal solution? Which constraits are binding and not binding?

Challenge 1: Your restaurant is doing well and you discover you have enough money to do one of the following: increase storage space so that you can store 100 lemons, purchase cups in packs of 24 instead of 18, or fix the ice machine so that it goes back to its original capacity of 44 scoops/hr. What should you do?

Challenge 2: Instead of fixing the ice machine, you decided to buy an entirely new one. The new machine makes enough ice for between 50 and 60 scoops of ice per hour. To make up for the money that you spent on the ice machine, you decide to increase the price of Charged Lemonades. You think you can sell the Charged Lemonades for between \$1.50 and \$3.00. Reformulate the Bread Company problem to maximize profit given this new information.