

A mathematical model applied to investigate the potential impact of global warming on marine ecosystems



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ABSTRACT

A mathematical model with time-varying parameters is newly proposed to describe the potential effects of rapid global warming on marine ecosystems. The time-varying parameters are assumed to vary in time, for example, environmental factors or a change in marine organisms. The existence of the proposed model is verified, as well as the stability of each equilibrium point is investigated. Beside, simulations, as well as a case study, are also carried out to justify the findings of this study comparing with others. It is found that global warming, because of the rapid concentration of greenhouse gases (GHGs), is leading marine ecosystems to an imbalanced situation by reducing significantly the marine plankton and fishery resources. Moreover, it is shown that if the density of marine plankton or its ability to absorb carbon dioxide (CO_2) can be increased, it can improve marine fishery resources by reducing global warming. Beside, this study could explain how marine ecosystems may change in the future as a result of rapid global warming. This study predicts that if the present situation of rapid global warming continues for the next 50 years unabated, it can damage marine ecosystems especially fishery resources in the long run. Overall, this study establishes a mathematical relationship between the environment and marine ecosystems that could contribute to environmental and fisheries management.

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1. Introduction

1.1. Motivation

Marine ecosystems are considered to be the largest producers of oxygen (O_2) and the largest absorbers of carbon dioxide (CO_2). Although CO_2 is an essential element in the photosynthesis of phytoplankton, the excess concentration of greenhouse gases (GHGs) is responsible for global warming which is causing rapid warming in the oceans. The warming is very destructive for planktonic organisms as well as fishery resources of marine ecosystems since it hurts the bicarbonate buffer of the ocean that keeps the ocean's acidity within the range of pH from 7.5 to 8.4 [1]. But the concentration of CO_2 in the atmosphere is constantly increasing and if this situation continues, the concentration of CO_2 will increase manifold by the end of this century as represented in Fig. A1 (Appendix A) [2]. Thus, if the concentration of CO_2 continues to increase in this way, the atmospheric temperature will rise proportionately and at the end of the century, it will increase manifold as shown in Fig. A2 (Appendix A) [2]. As a result of rapid global warming, the oceans are creating a hostile environment for

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which plankton and fish populations in marine ecosystems are steadily declining [3,4]. The rapid decline in marine plankton and fish populations is shown in Figs. A3 and A4 (Appendix A), respectively. If global warming continues in this way, a large part of marine ecosystems could be destroyed or depleted by the end of the present century.

1.2. Challenges

The concentration level of atmospheric CO₂ has been increasing significantly since the industrial revolution. The mean level of CO₂ has risen from 280.01 parts per million by volume (p.p.m.v.) to more than 380 p.p.m.v. [5,6] which breaks the history of the past 0.8 million years [5]. According to environmental scientists, the level of CO₂ will reach approximately 788.5 p.p.m.v. by the end of the 21st century if the present situation continues [2]. On the other hand, human beings are continuously amplifying the emissions of GHGs by discovering fuel-burning machines, vehicles, and aircraft. Despite knowing that the forest area and aquaculture are the main absorbers of atmospheric GHGs, human beings are indiscriminately cutting down forest areas and throwing or releasing toxic wastes (e.g., industrial waste, plastic, microplastics, sediments, and pesticides) into rivers and oceans which are dramatically damaging aquatic ecosystems [7–11].

Atmospheric temperature is rising proportionally with the increasing CO₂ concentration and the atmospheric temperature has increased approximately 0.85 °C by 1980–2012 which breaks the average rising rate (~0.2 °C/decade) [12]. Scientists predict that at the current rate of rising, warming may reach more than 4 °C by the end of the present century [2,5,13]. In this case, global warming, as well as climate change, can confine the normal lifestyle of all living beings on the earth by introducing different destructive natural phenomena. For example, global warming contributes to ocean acidification by the reduction of pH level hurting the bicarbonate buffer of the ocean [14]. Beside, some studies have shown that atmospheric CO₂ can directly cause acidity in ocean water like global warming, but it is less responsible for ocean acidity than global warming [15,16]. As a result of rapid global warming and CO₂ concentration, the pH level in the ocean's water has declined 0.1 pH units since 1980 whereas the decreasing rate of pH level was almost ~0.02 pH units/decade [17]. Ocean acidification badly influences marine ecosystems as well as coastal biodiversity by declining the nutrients and CaCO₂ and rising the hydrogenation [18].

Marine ecosystems play a significant role to balance the imbalanced environment by redistributing energy, heat, and materials. For example, marine ecosystems act as a sink for CO₂ which slows the rapid global warming as well as climate change, act as a source for O₂ (as about 70% O₂ is produced by phytoplankton [19]), and deal as a supplier of protein and vitamin to human beings [20–22]. Though the temperature is very essential for the photosynthesis of phytoplankton, the temperature which crosses the optimum level declines the photosynthesis and decreases the density of marine plankton [23]. Global warming mostly disrupts the growth of the seasonal successions of the phytoplankton in marine biodiversity [24]. It slowly breaks the reproduction capacity of plankton biomass by changing the turnover period of a species which thereby exerts pressure on the marine fishery community in the long run [25]. Overall, the effects of rapid global warming and GHGs concentration are greatly hindering the growth of marine phytoplankton, fishery resources, as well as marine algae [17,23,26,27] by introducing acidification and warming in the ocean's water. Meanwhile, some temperature-sensitive fish species are already on the verge of extinction [28]. According to a report [29], global warming has reduced the amount of marine fish (such as coral reefs and tuna) as well as phytoplankton in the Indian Ocean by about 20% over the last six decades. Scientists predict that if the situation continues, plankton and fish populations will be reduced to between 50 and 90% of current levels in the next five decades which will carry the marine ecosystem into an ecological desert and will no longer be productive [29]. By the end of the 21st century, the temperature of the tropical Pacific will rise more than 3 °C that can dramatically reduce marine biodiversity by threatening 50–80% of marine species, especially plankton [30]. Beside, the impact of climate change is continuously decreasing most of the valuable species in Canada's Pacific ecosystem, the Northwest Pacific Ocean, the Yellow Sea, and the East China Sea [31,32].

1.3. Background

There is a lot of works in literature focusing on the impacts of global climate change on coastal biodiversity as well as marine ecosystems [30,33–36]. Beside, several articles statistically described the potential impacts of global warming on marine ecosystems and coastal aquatic biodiversity [28,37–39]. The authors of those studies shown statistically and literary that the effects of global warming are continually damaging the overall ecosystem and if this situation continues, about 80 to 90% of the biodiversity in marine ecosystems could be damaged by the end of the century. For example, Asch et al. [30] theoretically described the effects of global warming on the aquatic biodiversity of the Pacific Ocean. The authors showed that if global warming continues in this way, atmospheric temperatures will rise by at least 3 °C by the end of the century, which could severely damage 50 to 74% of the aquatic biodiversity by lowering dissolved oxygen and pH levels in seawater. Christensen [36] conducted a systematic review of the literature over the period 1950–2018 to describe the effects of global warming on the marine ecosystems and fisheries resources of the Arabian Gulf. He observed that global warming is rapidly depleting marine organisms and fisheries, and it could bring the aquatic ecosystems of the Arabian Gulf to the ecological desert by the end of the current century. Speers et al. [28] illustrated the harmful impacts of global warming and acidification on marine fishery resources, especially shellfish and coral reefs. They found after statistical estimation that about 92% of oceanic reef fish could lose their existence by the end of 2100. But what or how much will change in the density of marine species with the change of any environmental factor is not described in those studies.

Table 1

A short review of some previous related studies [44].

References	Parameters (growth rate, decay rate,)		Impact of global warming on marine ecosystems			Solution method	Solution type			Case study
	Secondary	Estimated / Calculated	Global warming	Marine plankton	Marine fisheries		Exact	Heuristic	Meta-heuristic	
Sekerci and Petrovskii [19]		✓	✓	✓		DMM-based	✓			
Mandal et al. [21]		✓	✓	✓		DMM-based	✓			
Speers et al. [28]	✓		✓		✓	SBA	✓			
Asch et al. [30]	✓		✓	✓	✓	LBI			✓	
Häder and Barnes [33]			✓	✓		LBI			✓	
Baltar et al. [34]			✓	✓	✓	LBI		✓		
Mclean et al. [35]	✓		✓		✓	LBI		✓		
Christensen [36]			✓	✓	✓	LBI			✓	
Brierley and Kingsford [37]	✓		✓	✓	✓	SBA	✓			
Chapman et al. [38]	✓		✓	✓	✓	SBA	✓			
Moullec et al. [39]	✓		✓		✓	SBA		✓		
Hinners et al. [40]	✓		✓	✓		DMM-based	✓			
Kim and Kim [43]	✓		✓	✓		SMM-based	✓			
This paper		✓	✓	✓	✓	DMM-based	✓		✓	

A few articles analytically described the impacts of global warming on specific species of marine ecosystems through mathematical modeling. For example, Hinners et al. [40] developed a deterministic model to analyze the effect of global warming on marine phytoplankton and they found that the growth rate of marine phytoplankton is inversely changed with global warming. Sekerci and Petrovskii [19] analytically illustrated the effect of saturated CO₂ on marine plankton and found that the growth of marine plankton increases proportionally with the increasingly saturated CO₂ in the oceans. But the excess saturated CO₂ can demolish the zooplankton as well as the fishery resources in the ocean by introducing acidification and deficiency of saturated O₂ [14,16]. Mandal et al. [21] analytically analyzed the effect of atmospheric temperature on the growth of marine phytoplankton by developing a deterministic simulation-based model. They obtained that the density of marine plankton is much higher at the bottom of the ocean where the temperature reaches the optimum temperature (25 °C). But as the temperature exceeds the optimum temperature, the density of plankton gradually decreases. However, there are only a handful of works on mathematical modeling to describe the impact of climate change on marine species, except [41,42]. Overall, there is still a lacking of understanding mathematically of how the rapid concentration of GHGs changes the earth's climate as well as global warming and marine ecosystems. An extensive alignment of some previous researches mostly related to this study based on the characteristics, applications, and solution techniques is briefly presented in Table 1. The alignment emerges on the following distinguishing factors.

Parameters collection

Some studies [28,30,35,37–40,43] were carried out based on secondary parameters that are collected directly from various research articles, research literature, governmental or non-governmental websites. Other studies [19,21] were performed based on the estimated/calculated parameters. Here estimated parameters mean the parameters obtained after statistically analyzing the corresponding secondary parameters and/or data.

Impact of global warming on marine ecosystems

Global warming as well as climate change badly impacts marine ecosystems along with coastal biodiversity. Several authors described the impact of frequent global warming on the growth of only marine plankton as well as marine phytoplankton [19,21,33,40,43] or only marine fishery resources [28,35,39] or both of them [30,34,36–38].

Solution method

The solution method is another major aspect to solve real-life problems in research papers. To describe the impact of global warming on marine ecosystems, Sekerci and Petrovskii [19], Mandal et al. [21], Hinners et al. [40] proposed a deterministic mathematical model (DMM) whereas Kim and Kim [43] proposed a stochastic mathematical model (SMM). In some articles, the authors performed their works adopting statistical-based analysis (SBA) [28,37–39], and others illustrated their studies through literature-based analysis (LBA) [30,33–36].

Solution type

Exact solution methods are more efficient than any other solutions having minor errors to solve the real system. Sekerci and Petrovskii [19], Mandal et al. [21], Speers et al. [28], Brierley and Kingsford [37], Chapman et al. [38], Kim and Kim [43], and Hinners et al. [40] adopted exact methods to solve their formulated problems. In some other studies, the authors solved their problems using heuristic methods like sample average approximation [34,35,39]. Some researchers applied the meta-heuristic method to solve their problems [30,33,36].

Case study

Table 2
Description of parameters and their corresponding values used in this study.

Symbol	Descriptions	Values	References
r_1	Natural increasing rate of GHGs in the oceans	0.00095 kg km ⁻²	Estimated [4,17,34]
φ_1	Producing rate of GHGs by planktonic population in oceans	0.0029 kg km ⁻²	Estimated [15,21,29]
φ_2	Absorbing rate of GHGs by planktonic population in oceans	0.00099 kg km ⁻²	Estimated [4,21,37]
φ_3	Producing rate of GHGs due to global warming	1.0 μ kg km ⁻²	Estimated [15,17]
r_2	Natural growth rate of the atmospheric temperature	0.099 °C	Estimated [15,17]
σ_1	Rising rate of atmospheric temperature due to GHGs	0.00025 °C	Estimated [15,17]
σ_2	Absorbing rate of temperature by planktonic population	0.00565 °C	Estimated [17,21,37]
r_3	Normal growth rate of planktonic population	0.00225 km ⁻³	Estimated [3,4,21]
μ_1	Growth rate of the planktonic population due to CO ₂	0.00108 km ⁻³	Estimated [17,21,40]
μ_2	Hampering rate of plankton because of global warming	0.00001 km ⁻³	Estimated [15,21,40]
μ_3	Predation/consuming rate of plankton by fish population	0.0031 km ⁻³	Estimated [3,4]
μ_4	Losing rate of plankton by the effect of acidity	10.1 μ km ⁻³	Estimated [1,5,26,28]
r_4	Normal growth rate of fish population	0.0002/1000	Estimated [3,4]
η_1	Growth rate of fish populations by feeding/consuming plankton	175 μ /1000	Estimated [3,4]
η_2	Declining rate of fishes by the effect of acidity due to GHGs	190 μ /1000	Estimated [1,28,37]
η_3	Hampering rate of fish populations by global warming	61 μ /1000	Estimated [1,26,28]
a	Saturation constant	0.01	Estimated [5,17,21]
K_1	Carrying capacity of planktonic population	1000000 km ⁻³	Estimated [3,4,21]
K_2	Carrying capacity of fish population	10, 000 km ⁻³	Estimated [3,4]

We have conducted a case study to verify the validity of the model proposed in this study which has not been done in any of the other studies as mentioned in Table 1.

1.4. Objectives

The aim of this study is to analyze an environment management model with time-varying parameters that may take functional form in time. Here, environmental factors (such as CO₂ concentration) which contribute to global warming and affect marine ecosystems have been considered as time-varying parameters because they may change over time. In order to describe the effects of global warming caused by excess GHGs emissions on marine ecosystems (plankton diversity and fishery resources), we formulate a new mathematical model considering the concentration of GHGs, atmospheric temperature (global warming), marine plankton, and marine fishery resources as state variables. The model consists of a system of Nonlinear Ordinary Differential Equations (NODEs). This approach to modeling nonlinear interaction between environmental factors (GHGs and global warming) and species in marine ecosystems (plankton and fishery) has not yet been studied mathematically, and so, the study prolongs the analytic methods to deal with the considered time-varying parameters. Also, how much or what will change in plankton and fish populations in marine biodiversity with the change of global warming is described in this paper. Beside, this study numerically analyzes the dynamic behavior of the dynamic species for the long term to predict the future of marine ecosystems (plankton and fish populations) along with rapid global warming.

2. Methods and materials

2.1. Workflow of the study

The fight of living beings against their environment is a common natural phenomenon. Since the industrial revolution, the emissions of environmental GHGs are significantly increasing [2,5,6]. With the increasing concentration of GHGs, global warming, as well as the earth's climate, is continuously changed. It mainly damages marine ecosystems by introducing acidification, warming, and natural calamities (such as floods, tsunami, etc.) [15,23,26–32]. As a result, it leads the environment to an unbalanced situation. Several authors conducted their studies focusing on the impacts of global warming and climate change on marine ecosystems [15,23,28–39,45], but none analyzed how much changes will occur in the growth of plankton and fish populations in marine ecosystems if any environmental factor (such as emissions of GHGs, rising of global warming, etc.) change with time, as described in Section 1 (Section 1.3). By finding the research gap, we searched for the interrelationships among the concentration of GHGs, global warming, and marine ecosystems, and obtained a set of parameters used in this study by parametric estimation, presented in Table 2 and explicitly described in Section 1 (Section 1.3). After that, we have proposed a new mathematical model in Section 2 (Section 2.2) to fill the research gap which describes the impacts of global warming on marine ecosystems and predicts the future marine ecosystems if the present situation continues. In Appendices B–D, we have analyzed the existence theorem for dynamic species and the stability analysis has been performed at each equilibrium point to obtain the dynamic behavior of the proposed model. In Section 3, the findings and application of this study are disclosed through simulations, case studies, and bifurcation analysis. Finally, some conclusions, limitations, and future directions of the study are presented in Section 4. To better understand the processes of this study, a workflow diagram is briefly presented in Fig. 1.

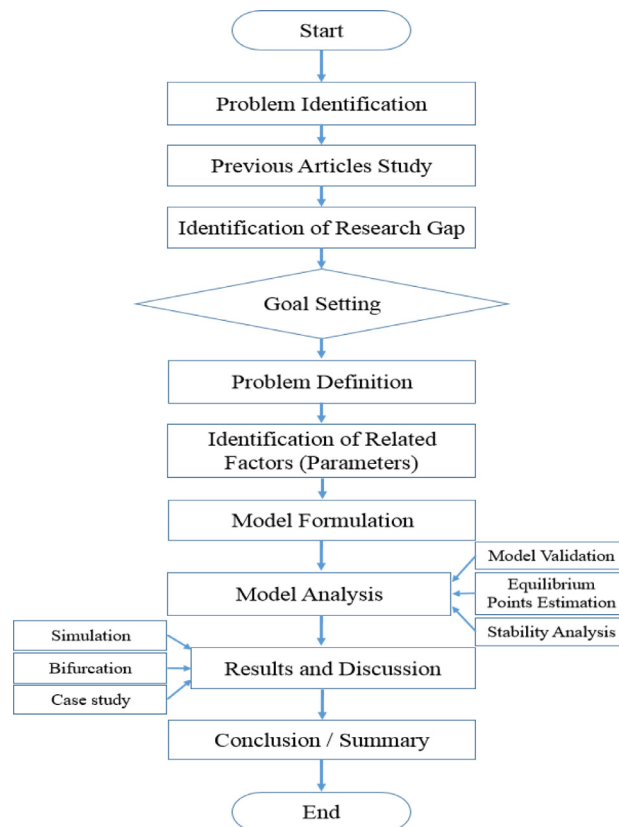


Fig. 1. Workflow diagram of this study.

2.2. Model formulation

The objective of this study is to analyze the impact of global warming due to the rapid concentration of GHGs on marine ecosystems (plankton and fish populations) and also to predict the future of marine ecosystems with the rapid global warming. In this regard, we have employed a multi-species deterministic mathematical model since mathematical modeling is greatly used to analyzed natural phenomena [13,19,21,40]. The heterogeneous system is split into four distinct categories: the concentration of environmental GHGs, $G(t)$, which are rapidly emitted from various sources (mostly by different activities of human beings and slowly by natural phenomena) [2]; the rising atmospheric temperature, $T(t)$, which is rising proportionally with the increasing concentration of environmental GHGs and is responsible for global warming [2,12]; the density of planktonic population in marine ecosystems, $P(t)$, which is continuously threatened by the rapid warming and GHGs concentration [15,23,24,26–32]; and the density of fish population in marine ecosystems, $F(t)$, which is also declined in the amount due to rapid warming, acidification, deficiency of saturated oxygen and deficiency of planktonic population [15,23,24,26–32]. Fig. 2 presents the schematic diagram of the model briefly illustrating the impacts of rapid global warming and GHGs concentration on marine ecosystems.

Various chemical reactions occur among environmental gases in the presence of UV radiations. In the reaction period, they emit heat which raises the atmospheric temperature and produces some number of new GHGs components which naturally increase the concentration of environmental GHGs [46]. Here r_1 and r_2 are the normal growth rate of $G(t)$ and $T(t)$, respectively. On the other hand, r_3 and r_4 are respectively the normal growth rate of $P(t)$ and $F(t)$ in absence of harmful impacts of GHGs and global warming. In marine ecosystems, plankton (phytoplankton) absorbs the environmental CO_2 for photosynthesis which declines the concentration of environmental GHGs [19,23]. The absorption of environmental GHGs by the planktonic population is denoted by φ_2PG . Conversely, the fish population in marine ecosystems emits saturated CO_2 which slightly increases the concentration of GHGs. Here, φ_1FG presents the increase in GHGs concentration by the fish population. On the other hand, natural destruction due to global warming such as drought and forest fire promotes GHGs concentration [47]. Here, φ_3T denotes the production of GHGs due to global warming.

The atmospheric temperature proportionally increases with the concentration of environmental GHGs [2,5,12,13]. Here σ_1GT presents the increase in atmospheric temperature due to the increasing GHGs. Temperature is an essential factor for the photosynthesis of marine plankton (phytoplankton) [19,23]. Thus, the marine planktonic population can slow down the rising temperature by photosynthesis. Here, σ_2PT denotes the absorption of atmospheric temperature by planktonic

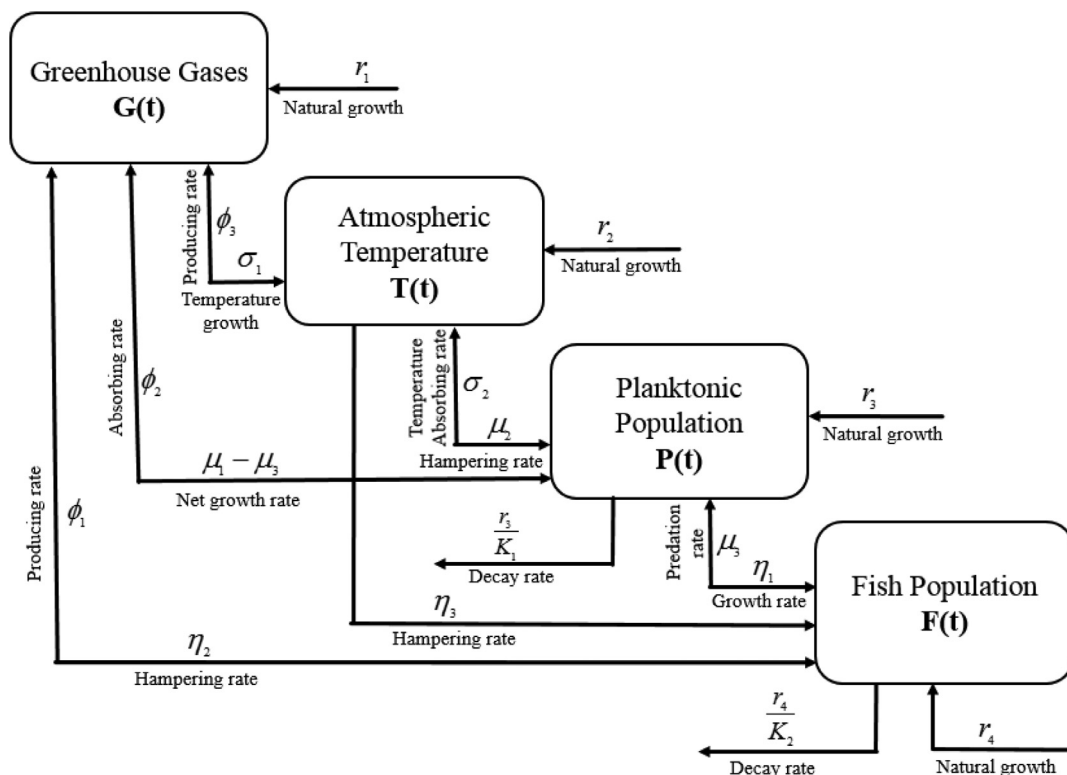


Fig. 2. The schematic diagram of the model (1–4) briefly describing the impacts of GHGs on global warming and marine ecosystems.

population. We assume that the saturated constant of dissolved CO_2 is $a(0 < a < 1)$ which is a constant. Since all living organisms have a maximum carrying capacity to lead their life-cycle [25], we consider K_1 and K_2 are the carrying capacity of the planktonic population and fish population, respectively, where $\frac{r_3}{K_1}$ and $\frac{r_4}{K_2}$ are the corresponding decay rates. Though the concentration of dissolved CO_2 promotes the density of marine plankton, the growth of the planktonic population becomes slow when the density of dissolved CO_2 is so high because it hampers the plankton’s respiration by declining the density of dissolved O_2 . Because of the deficiency of dissolved O_2 and the influence of saturated CO_2 , it hampers the growth of marine fishery [15,23,26,27]. Therefore, $\frac{\mu_1 P}{a+G}$ represents the increase in planktonic population by absorbing CO_2 whereas $\frac{\eta_2 F}{a+G}$ denotes the decrease in fish population due to high concentration of dissolved CO_2 . Warming and acidification in the ocean’s water are very harmful to marine ecosystems by which the density of marine plankton, as well as fishes, are continuously decreasing [15,23,24,26–32]. Here $\mu_2 TP$ presents the decline in planktonic population due to warming, $\mu_4 GP$ is the decrease in planktonic population for the effect of acidity, and $\eta_3 TF$ denotes the decrease in fish population due to the effect of global warming. In marine ecosystems, the planktonic population acts as prey, and the fish population acts as predators [30,37,38]. So, we assume that $\mu_3 FP$ is the decrease in the planktonic population due to the predation/consumption of fish population and $\eta_1 PF$ is the increase in fish population by consuming the planktonic population. The parametric descriptions along with corresponding values are briefly represented in Table 2.

According to the above assumptions, Fig. 2, and conservation and balance principle [48], we newly formulate the following mathematical model consisting of a system of NODEs:

$$\frac{dG}{dt} = r_1 G + \phi_1 F G - \phi_2 P G + \phi_3 T \tag{1}$$

$$\frac{dT}{dt} = r_2 T + \sigma_1 G T - \sigma_2 P T \tag{2}$$

$$\frac{dP}{dt} = r_3 P \left(1 - \frac{P}{K_1}\right) + \frac{\mu_1 P}{a+G} - \mu_2 T P - \mu_3 F P - \mu_4 G P \tag{3}$$

$$\frac{dF}{dt} = r_4 F \left(1 - \frac{F}{K_2}\right) + \eta_1 P F - \frac{\eta_2 F}{a+G} - \eta_3 T F \tag{4}$$

with initial conditions $G_0 = G(0) > 0$, $T_0 = T(0) > 0$, $P_0 = P(0) \geq 0$, $F_0 = F(0) \geq 0$.

The detailed analytical analysis validating the model has been given in Appendices B–D.

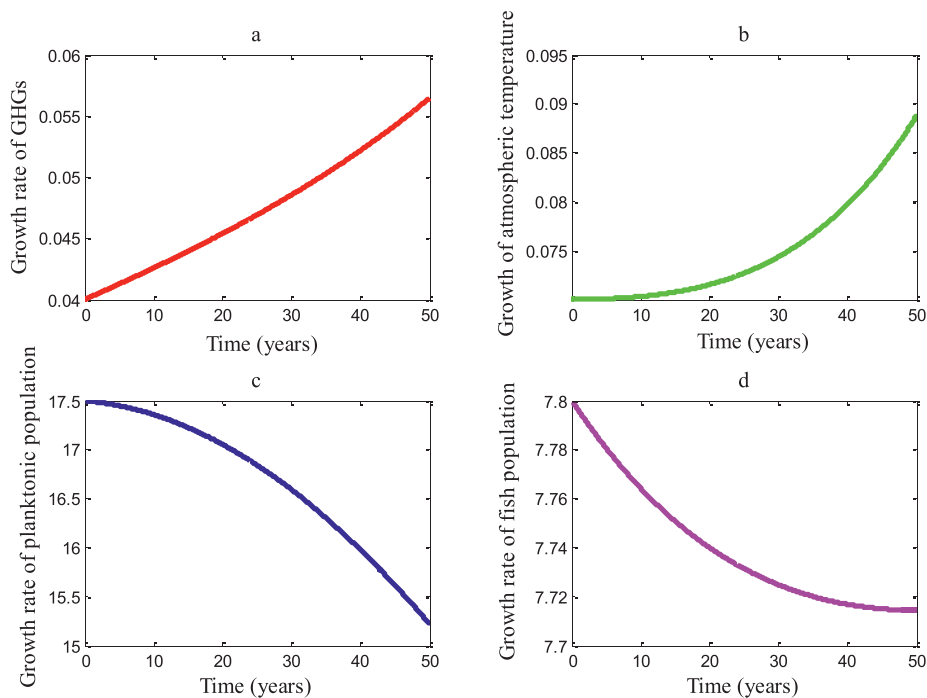


Fig. 3. Effects of increasing GHGs on atmospheric temperature and marine ecosystems.

3. Results and discussion

The numerical simulations of the model (1)–(4) have been performed using ode45 solver in MATLAB programming language and setting the parametric values from Table 2 whereas the initial values of the dynamic species are $G_0 = 0.04$, $T_0 = 0.07$, $P_0 = 17.5$, $F_0 = 7.8$. For the computational analysis, we have used the estimated parametric values whereas the related data are collected from secondary sources [1,3–5,15,17,21,26,28,29,34,37,40]. The simulations aim to verify the analytical results of this study and also to describe the dynamic behaviors of the considered species especially the plankton and fish populations in marine ecosystems under rapid global warming.

Now, we are going to describe the potential impact of global warming due to the rapid concentration of GHGs on the planktonic population and fishery resources in marine ecosystems. The concentration of environmental GHGs is continuously increasing, consequently, the atmospheric temperature is proportionally increasing with the rapid GHGs concentration. The rapid concentration of GHGs contributes to introducing acidification in the ocean's water which destroys both plankton diversity and fishery resources in oceans. Beside, marine fishery resources are being threatened and decreased mainly due to rapid global warming. On the other hand, the fish population proportionally decreases with the decline in plankton diversity because of food scarcity. Therefore, the increasing concentration of GHGs is promoting global warming which dramatically decreases the planktonic population by introducing acidity and warming, and their corresponding results are significantly declining fishery resources. The changes in the growth of the dynamic species are represented in Fig. 3.

Now, we are going to describe the dynamic behaviors of the considered dynamic species for different absorbing rates of GHGs by the planktonic population (φ_2) which are represented in Figs. 4–7. When the absorbing rate of saturated CO_2 and nitrous oxide (N_2O) by the planktonic population increases, the increasing concentration of GHGs slows as shown in Fig. 4. The increasing absorption of environmental GHGs by the planktonic population contributes to decreasing the atmospheric temperature since the atmospheric temperature is proportionally changed with the concentration of environmental GHGs as shown in Fig. 5. CO_2 is essential for the photosynthesis of the plankton population, and N_2O is essential in preserving O_2 for respiration. Therefore, the growth rate of the marine plankton grows up when the absorption of saturated CO_2 and N_2O are increased. Fig. 6 represents the improving rate of the planktonic population when the absorption of GHGs increases. On the other hand, since the fish population consumes the planktonic population, so their growth is proportionate to the plankton population. When the planktonic population rises by absorbing dissolved CO_2 , the fish population also increases by getting sufficient food from plankton, getting sufficient dissolved O_2 , and lower warming in the ocean's water as presented in Fig. 7. The simulations conclude that when the absorbing rate of GHGs by the marine plankton increases, it declines global warming by reducing GHGs concentration and improves the density of plankton diversity along with the fishery resources.

Here, we are going to illustrate the changes in the growth of the dynamic species when the growth of the marine plankton varies which are represented in Figs. 8–11. Similarly, when the plankton density increases in marine ecosystems, it

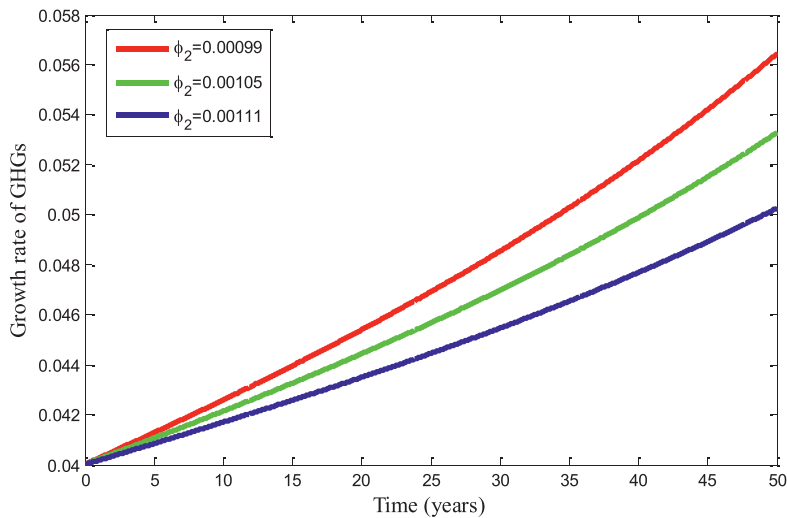


Fig. 4. The decreasing rate in the concentration of GHGs due to the increasing values of ϕ_2 .

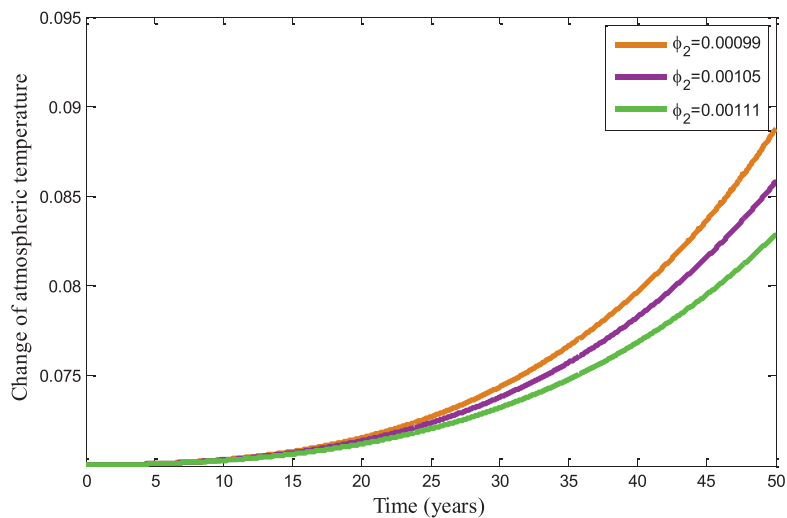


Fig. 5. The decreasing rate of the atmospheric temperature due to the increasing values of ϕ_2 .

increases the absorption of environmental CO₂. As a result, the concentration of environmental GHGs comes down with the increasing plankton density as presented in Fig. 8. As usual, the atmospheric temperature proportionally decreases due to the decrease in GHGs concentration because of the increasing plankton population as displayed in Fig. 9. When the dissolved CO₂ in the ocean’s water is available enough, the planktonic population can easily perform their photosynthetic activities that improve the plankton population. The growth of the planktonic population is shown in Fig. 10. As similar as previously, the growth of the fish population proportionally increases with the increasing plankton diversity because of getting more food and lower warming which is shown in Fig. 11. Therefore, Figs. 8–11 represents that if the density of marine plankton can be increased, it may control global warming as well as the concentration of GHGs, and can also improve the fishery resources in the oceans.

Beside, the ocean’s water is being warm, acidity and deficiency of dissolved O₂ due to rapid global warming, consequently, the marine fishery resources are significantly decreasing day by day. Here, Fig. 12 represents that the marine fish population is steadily declining due to increasing global warming.

Bifurcation analysis

A qualitative change in the dynamic behavior of the system (1)–(4) can be represented by bifurcation analysis depending on the parametric variation. Therefore, the bifurcation diagram represents the dynamic changes of the equilibrium points concerning the variations of parametric values. In the case of bifurcation analysis of the system, the format of the Jacobian matrix $J \equiv DF(X)$ is needed to change to the new format $J \nu \equiv \lambda \nu$, where λ represents the set of eigenvalues and ν denotes

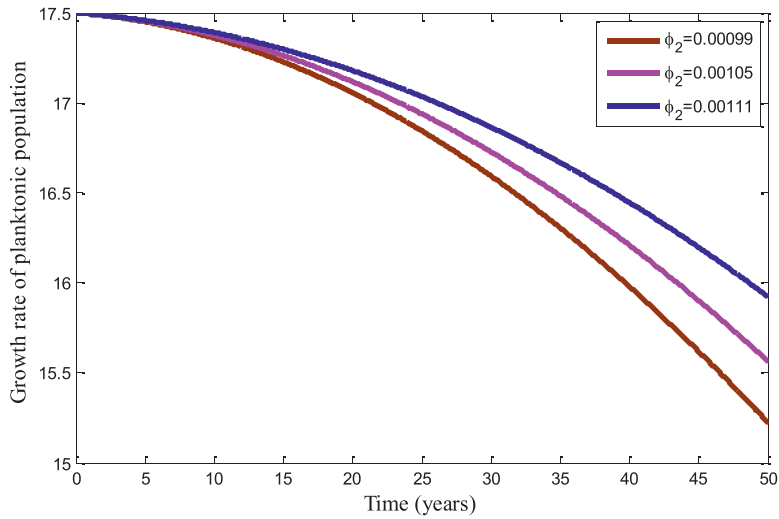


Fig. 6. The increasing rate of the planktonic population due to the increasing values of φ_2 .

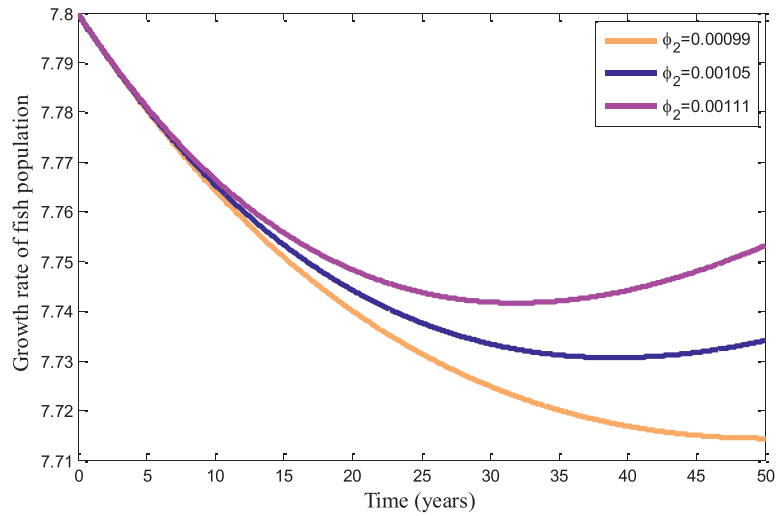


Fig. 7. The increasing rate of the fish population due to the increasing values of φ_2 .

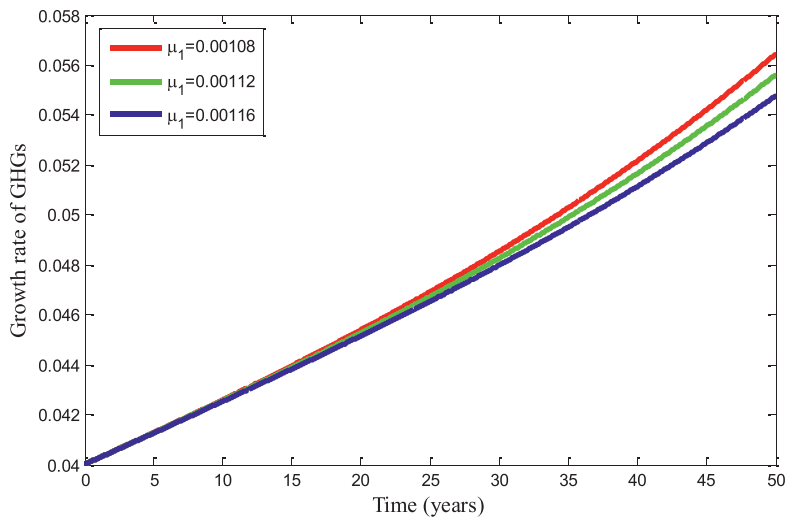


Fig. 8. The declining rate of the concentration of GHGs for the increasing values of μ_1 .

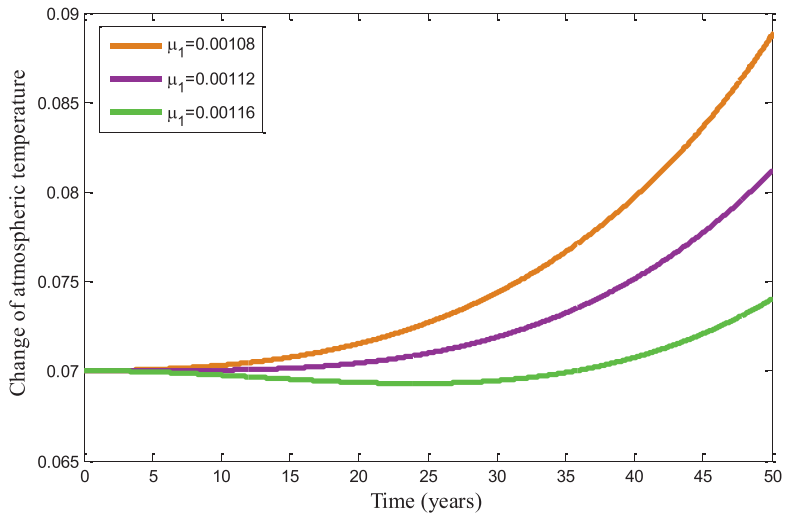


Fig. 9. The declining rate of global warming for the increasing values of μ_1 .

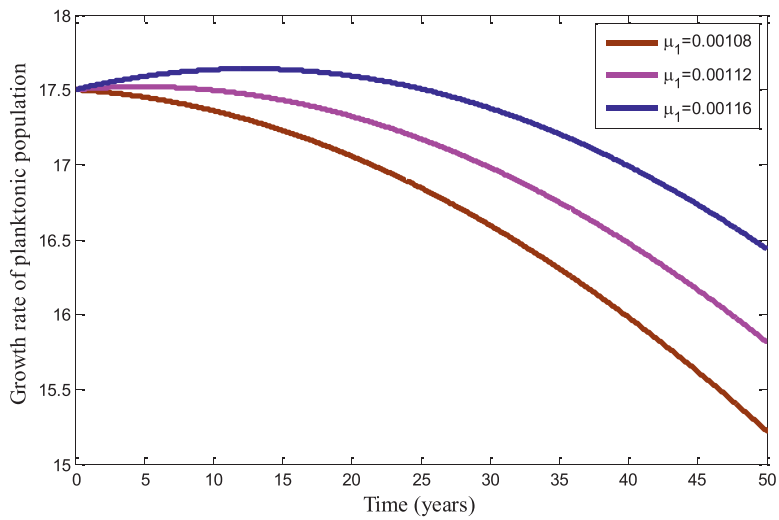


Fig. 10. The increasing rate of the planktonic population due to the increasing values of μ_1 .

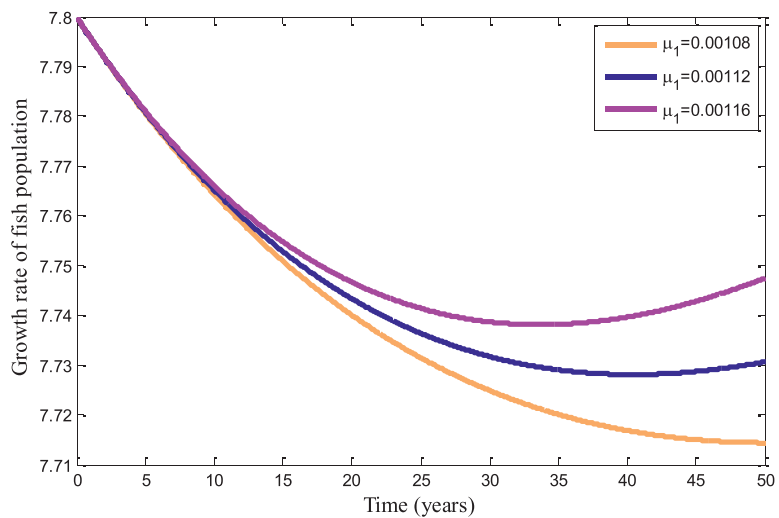


Fig. 11. Improving rate of the fish population due to the increasing values of μ_1 .

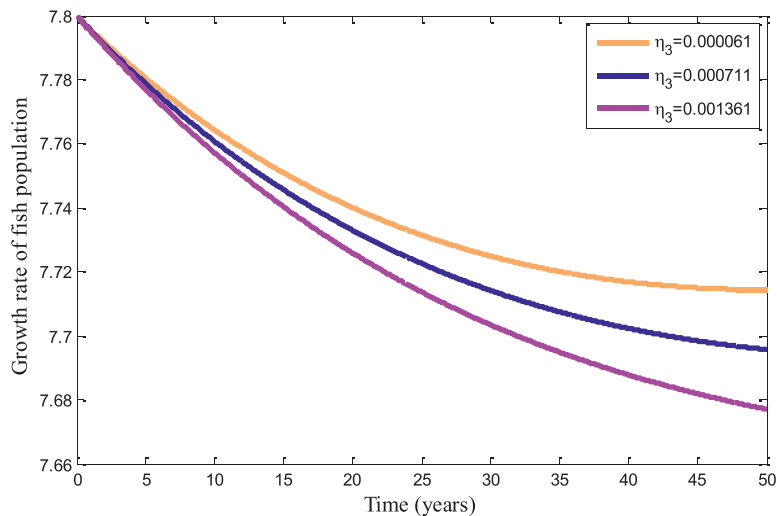


Fig. 12. The decreasing rate of the fish population due to increasing global warming (η_3).

Table 3

Growths of the considered dynamic species for the increasing absorbing rate of GHGs by the planktonic population over 50 years.

Growth rate of the considered species	Absorbing rates of GHGs by the planktonic population (φ_2)			Presented figures	Compared with the papers
	$\varphi_2 = 0.00099$	$\varphi_2 = 0.00105$	$\varphi_2 = 0.00111$		
Environmental GHGs, $G(t)$	0.0564	0.0532	0.0515	Fig. 4	[2,5,6,19,23]
Atmospheric temperature, $T(t)$	0.0889	0.0854	0.0826	Fig. 5	[2,5,12,13]
Planktonic population, $P(t)$	15.246	15.546	15.912	Fig. 6	[15,19,23,24,26–32,37,38]
Fish population, $F(t)$	7.7152	7.7345	7.7535	Fig. 7	[15,23,24,26–32,37,38]

the set of eigenvectors. However, at any equilibrium point $(\tilde{C}, \tilde{T}, \tilde{P}, \tilde{F})$, the Jacobian matrix becomes

$$\begin{bmatrix} r_1 + \varphi_1 \tilde{F} - \varphi_2 \tilde{P} & \varphi_3 & -\varphi_2 \tilde{C} & \varphi_1 \tilde{C} \\ \sigma_1 \tilde{T} & r_2 + \sigma_1 \tilde{C} - \sigma_2 \tilde{P} & -\sigma_2 \tilde{T} & 0 \\ -\frac{\mu_1 \tilde{P}}{(a+\tilde{C})^2} - \mu_4 \tilde{P} & -\mu_2 \tilde{P} & a_{33} & -\mu_3 \tilde{P} \\ \frac{\eta_2 \tilde{F}}{(a+\tilde{C})^2} & -\eta_3 \tilde{F} & \eta_1 \tilde{F} & a_{44} \end{bmatrix} \begin{bmatrix} v_C \\ v_C \\ v_P \\ v_F \end{bmatrix} = \lambda \begin{bmatrix} v_C \\ v_C \\ v_P \\ v_F \end{bmatrix}$$

Here, the eigenvalues λ disclose information on the strength and direction of the attraction and repulsion of the orbit. If λ take complex or real numbers, we have respectively a spiral node or a node only. Where the bifurcation occurs or not, and when the critical points will be stable, a node, a saddle point, or unstable, it can be easily found by using the eigenvalues and evaluating $tr(J)$ and $det(J)$ at the equilibrium points. The simulations are performed for the bifurcation analysis considering three dynamic species of the system (1)–(4) at a time. Here, Fig. 13 represents the bifurcation diagram of the system (1)–(4) consisting of four diagrams that illustrate the nature of the dynamic species. Fig. 13 shows that global warming and GHGs concentration are constantly increasing over time by which the overall marine ecosystems i.e. marine plankton and fisheries resources are continuously declining.

A case study

To verify the analytical results and to describe the application technique of the model (1)–(4), a case study is also conducted in this paper. We analyzed the adverse impacts of the rapid concentration of environmental GHGs on frequent global warming and marine ecosystems (plankton and fish populations) in this study. In this regard, we newly formulated a model (1)–(4) considering the concentration of GHGs, global warming, planktonic population, and fish population as the dynamic species. The model (1)–(4) is verified by analysis. The results of this study are also verified by comparing them with some other papers and natural phenomena, employing numerical simulations, which are represented in Tables 3 and 4.

This paper is carried based on the present situation, and numerically analyzed the model (1)–(4) by using current parametric values from Table 2. We predict the future growth of the dynamic species if the present situation continues for the next 50 years which is shown in Fig. 3. To compare the results before and after applying the model for a real case, we performed a comparison in Figs. 4–12.

Since the phytoplankton absorbs the saturated CO_2 for photosynthesis, the planktonic population contributes to absorbing CO_2 and the absorbing capacity of CO_2 proportionally depends on the density of the plankton population [23]. By removing the unsaturated waste such as polythene, plastic, and oil, the ocean’s water can be cleaned and toxic-free which can improve the plankton density and can control global warming and GHGs concentration by absorbing more CO_2 [7–11]. Therefore, if

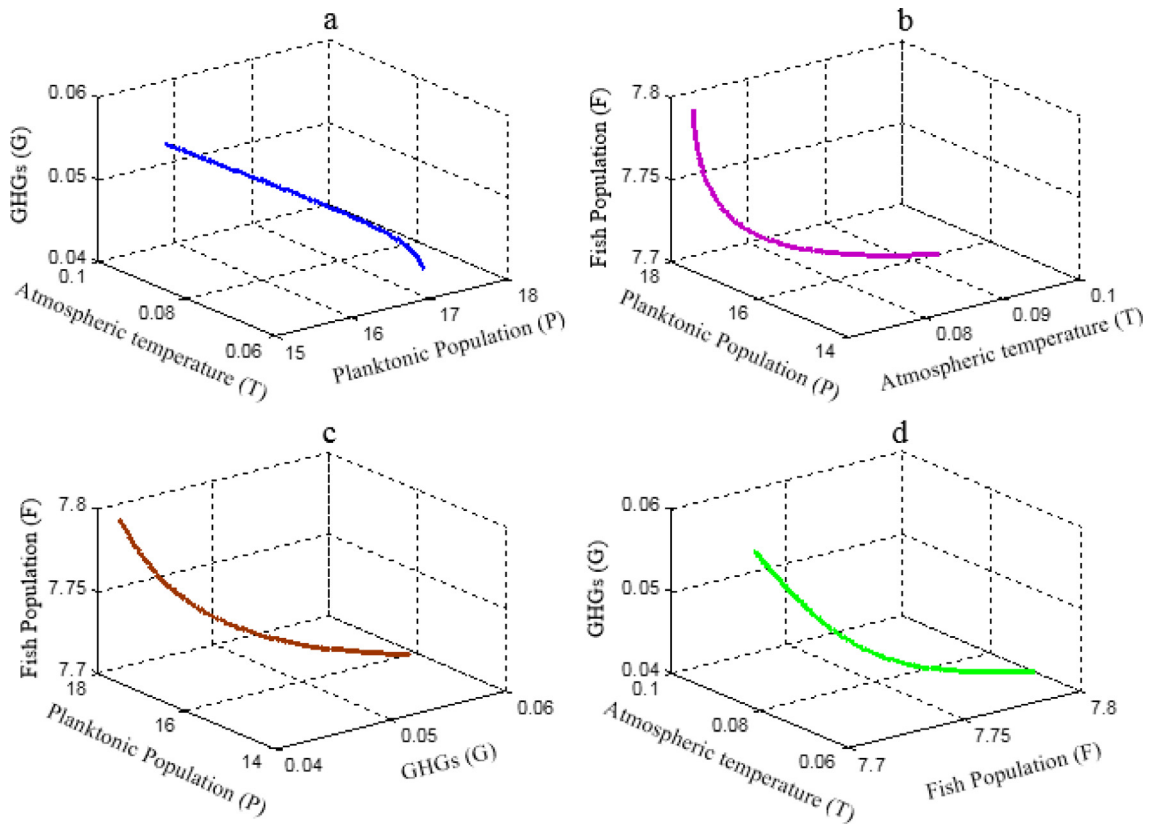


Fig. 13. Bifurcation diagram of the model (1–4) for the concentration of GHGs $G(t)$, atmospheric temperature $T(t)$, planktonic population $P(t)$, and fish population $F(t)$.

Table 4

Growth rates of the considered dynamic species for the increasing density of the planktonic population due to saturated CO_2 over 50 years.

Growth rate of the considered species	Growth rate of the planktonic population due to saturated CO_2 (μ_1)			Presented figures	Compared with the papers
	$\mu_1 = 0.00108$	$\mu_1 = 0.00112$	$\mu_1 = 0.00116$		
Environmental GHGs, $G(t)$	0.0564	0.0558	0.0548	Fig. 8	[2,5,6,19,23]
Atmospheric temperature, $T(t)$	0.0889	0.0817	0.0742	Fig. 9	[2,5,12,13]
Planktonic population, $P(t)$	15.246	15.821	16.465	Fig. 10	[15,19,23,24,26–32,37,38]
Fish population, $F(t)$	7.7152	7.7311	7.7478	Fig. 11	[15,23,24,26–32,37,38]

the absorbing rate of CO_2 by the planktonic population (φ_2) can be increased from 0.00099 to 0.00105 or 0.00111, the increasing rate of GHGs concentration will decline from 0.0564 to 0.0532 or 0.0515, respectively, in the next 50 years [19,23] which is shown in Fig. 4, consequently, it can slow the rising rate of the atmospheric temperature from 0.0889 to 0.0854 or 0.0826, respectively [19,23], as presented in Fig. 5. As a result, it can improve marine biodiversity by increasing the planktonic population and fish population [15,26–32,37,38] which are respectively presented in Figs. 6 and 7, and summarised in Table 3. Similarly, if we can improve the density of the marine planktonic population, it may come down the concentration of environmental GHGs [19,23] as shown in Fig. 8, may decline the rising of atmospheric temperature [19,23] as represented in Fig. 9, can recover the diversity of marine plankton [15,26–32,37,38] as displayed in Fig. 10 and can enhance the marine fishery resources [15,26–32,37,38] as shown in Fig. 11. The numerical investigations are summarised in Table 4. Inversely, if we are unable to control the concentration of environmental GHGs, warming and acidification in the ocean’s water can sharply damage marine biodiversity, especially fishery resources as presented in Fig. 12.

To describe the application of this study, we consider the following set of solutions of the model (1)–(4) (Appendix C)

$$\begin{cases} G \approx \frac{\varphi_3 T}{\varphi_2 P - r_1 - \varphi_1 F} \\ T \approx \frac{r_1}{\sigma_1 \varphi_3} (r_2 - \sigma_1 P) \\ P \approx \frac{\mu_3 \sigma_2 F}{2\sigma_1 \left(\frac{\mu_2 r_1 - r_3}{\varphi_3} - \frac{r_3}{K_1} \right)} + \frac{1-a}{2} - \frac{r_2}{2\sigma_1} \\ F \approx \frac{a\eta_1 K_1 (\sigma_1 r_3 K_1 + r_2 \mu_4 K_1)}{4r_3 r_4} \end{cases} \quad (5)$$

From the set of solutions (5), we can estimate the qualitative behavior of the concentration of GHGs, the rising of atmospheric temperature, the plankton and fish populations in marine ecosystems by using the parametric values from Table 2. If one or more environmental factors i.e. parametric values change over time, this study can also predict the future status of the considered dynamic species. These are the main application of this study in real problems. In this case, the solutions will remain unchanged unless there is a change in environmental factors, but since the parametric values of this study are mostly related to environmental factors, the solutions of this study may change when the related environmental factors change. It is noted that the algebraic equations of the system (5) have to be performed at the same time to obtain the solutions because they are naturally dependent on each other.

4. Conclusions

In this paper, we newly propose an environment management model (1)–(4) with time-varying parameters. The model investigates the impacts of global warming due to the rapid concentration of environmental GHGs on marine ecosystems. The model is verified by analysis and the analytical results of this study are examined by numerical simulations. We investigate for the first time how the marine ecosystems, as well as global warming, will be changed if there is a change in the concentration of environmental GHGs. In this regard, we have compared the growths of the atmospheric temperature, planktonic population, and fish population for the fluctuation of GHGs concentration. We find that global warming is increasing rapidly due to the rapid GHGs concentration which is increasing the warmth and acidity of ocean water, consequently, the effects are steadily declining the plankton and fish populations in marine ecosystems. Besides, we find that the marine fishery resources are proportionate to the marine plankton which is rapidly decreasing due to the decrease in plankton population. Simulations indicate that marine fishery resources can be enriched by improving marine plankton diversity along with reducing the concentration of GHGs. This study also predicts that if the present situation continues, marine ecosystems may turn into ecological deserts in the next 50 years due to the continuous decrease in plankton diversity and fishery resources.

This study is performed as a theoretical foundation in terms of mathematical equations for real scenarios based on environmental factors. Therefore, the results of this study may change in the future if the study-related environmental factors and marine organisms change.

For future exploration, this study can be carried by introducing a system of impulsive differential equations (IDEs) and comparing the results of ordinary differential equations (ODEs) and IDEs. For another future research, this model (1)–(4) can be extended to an optimal control method by introducing effective control strategies aiming to maximize the marine fishery resources by improving marine plankton and reducing GHGs concentration and global warming.

Declaration of Competing Interest

The authors have no conflict so that the publication of the manuscript can be interrupted.

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Appendix A

The present status of the dynamic species are given below:

[Figs. A1–A4.](#)

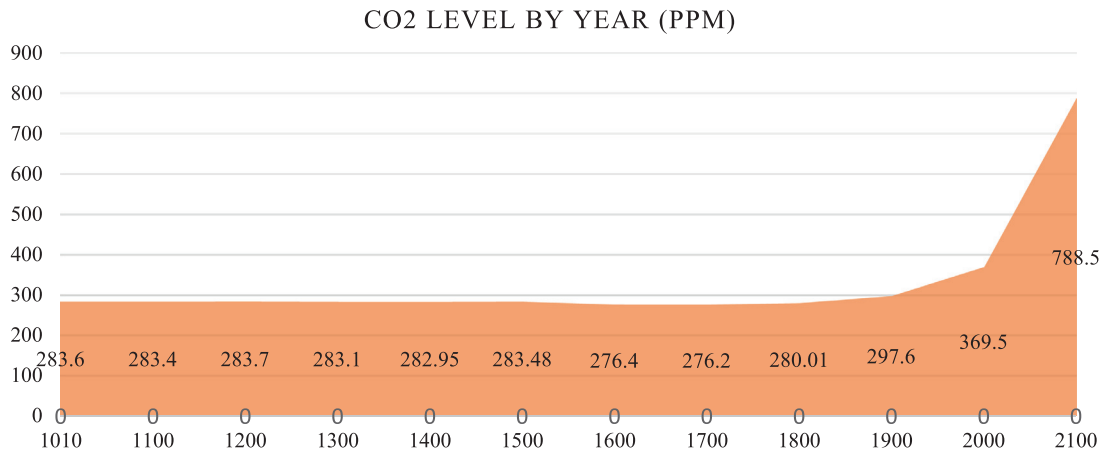


Fig. A1. The increasing rate of CO₂ in the atmosphere from 1010 to 2100 [2].

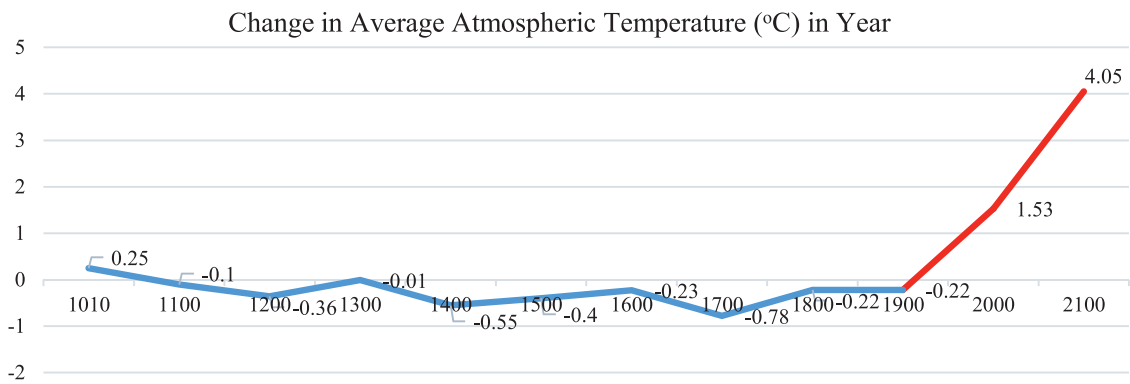


Fig. A2. The increasing rate of atmospheric temperature from 1010 to 2100 [2].

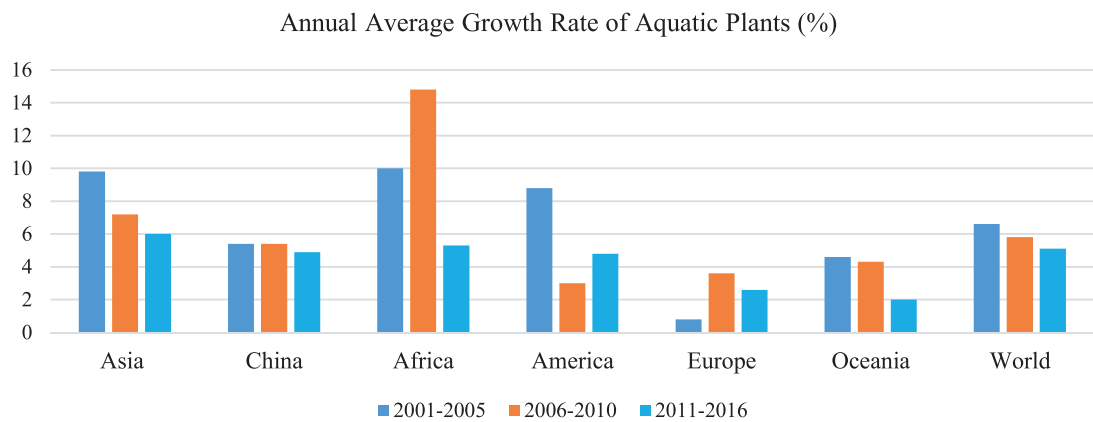


Fig. A3. Worldwide annual average growth of aquaculture production [3].

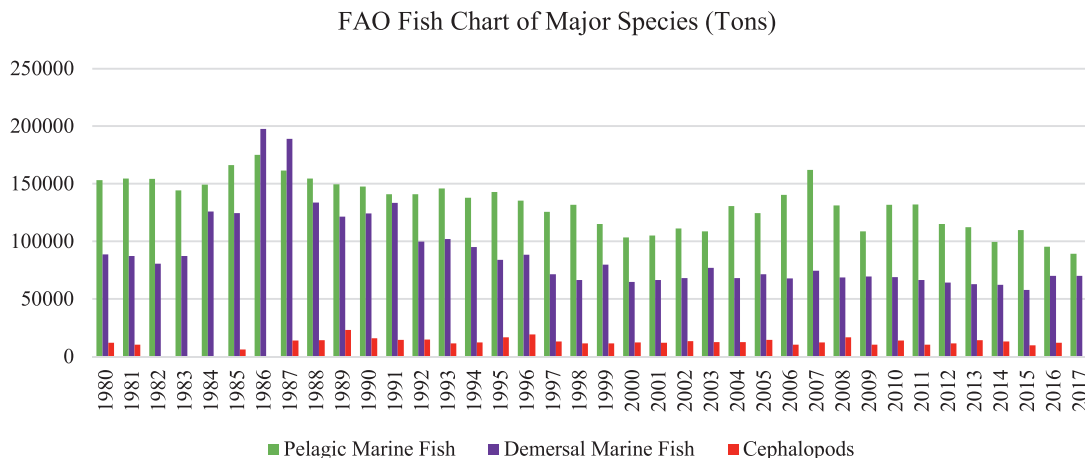


Fig. A4. The production of some major fish species for the Portuguese Republic [4].

Appendix B

The boundedness and existence theorems for the dynamic species of the model (1)–(4) have been analytically proved in the vein of [42,48–50].

Boundedness of the system

Now, we are going to establish that the solutions of the system (1)–(4) are bounded and non-negative for all $t \geq 0$.

Lemma 1. Let $\Omega = \{(G, T, P, F) \in \mathbb{R}_+^4 : g(t) = G(t) + T(t) + P(t) + F(t), 0 \leq g(t) \leq \frac{v}{w}\}$ be the region of attraction where w is a constant and $v = \frac{K_1}{4r_3}(r_3 + w)^2 + \frac{K_2}{4r_4}(r_4 + w)^2 + \frac{1}{8\mu_4}(r_1 + w)^2 + \frac{1}{4\eta_3}(\varphi_3 + r_2 + w)^2$ then the system (1)–(4) is positively bounded.

Proof. Let $g(t) = G(t) + T(t) + P(t) + F(t)$ be constant. Then from the condition, we can write

$$\begin{aligned} \frac{dg}{dt} + wg &= \frac{dG}{dt} + \frac{dT}{dt} + \frac{dP}{dt} + \frac{dF}{dt} + wG + wT + wP + wF \\ \Rightarrow \frac{dg}{dt} + wg &= (r_1 + w)G + (\varphi_3 + r_2 + w)T + (r_3 + w)P - \frac{r_3P^2}{K_1} + (r_4 + w)F - \frac{r_4F^2}{K_2} + (\varphi_1F - \varphi_2P)G + \\ &\quad (\sigma_1G - \sigma_2P)T + \left(\frac{\mu_1}{a+G} - \mu_2T - \mu_3F - \mu_4G\right)P + \left(\eta_1P - \frac{\eta_2}{a+G} - \eta_3T\right)F \end{aligned}$$

Since the growth rate of atmospheric temperature due to GHGs is not less than the absorbing rate of temperature by the planktonic population in the oceans i.e. $\sigma_1 \geq \sigma_2$. Whereas the producing rate of GHGs by fish population is very small compared to the absorbing rate by planktonic population i.e. $\varphi_2P \geq \varphi_1F$. However, assuming $(\mu_2T + \mu_3F - \mu_4G - \frac{\mu_1}{a+G})P \approx 2\mu_4G^2$ and $(\frac{\eta_2}{a+G} + \eta_3T - \eta_1P)F \approx \eta_3T^2$, we have

$$\begin{aligned} \Rightarrow \frac{dg}{dt} + wg &= (r_3 + w)P - \frac{r_3P^2}{K_1} + (r_4 + w)F - \frac{r_4F^2}{K_2} + (r_1 + w)G - 2\mu_4G^2 + (\varphi_3 + r_2 + w)T - \eta_3T^2 \\ \Rightarrow \frac{dg}{dt} + wg &= \frac{K_1}{4r_3}(r_3 + w)^2 - \frac{r_3}{K_1}\left(P - \frac{K_1}{2r_3}(r_3 + w)\right)^2 + \frac{K_2}{4r_4}(r_4 + w)^2 - \frac{r_4}{K_2}\left(F - \frac{K_2}{2r_4}(r_4 + w)\right)^2 + \frac{1}{8\mu_4}(r_1 + w)^2 \\ &\quad - 2\mu_4\left(G - \frac{r_1+w}{4\mu_4}\right)^2 + \frac{1}{4\eta_3}(\varphi_3 + r_2 + w)^2 - \eta_3\left(T - \frac{1}{2\eta_3}(\varphi_3 + r_2 + w)\right)^2 \\ \therefore \frac{dg}{dt} + wg &\leq \frac{K_1}{4r_3}(r_3 + w)^2 + \frac{K_2}{4r_4}(r_4 + w)^2 + \frac{1}{8\mu_4}(r_1 + w)^2 + \frac{1}{4\eta_3}(\varphi_3 + r_2 + w)^2 \end{aligned}$$

Applying the differential inequalities, we have

$$0 \leq g(G(t), T(t), P(t), F(t)) \leq \frac{v}{w}\left(1 - e^{-wt}\right) + (G(0) + T(0) + P(0) + F(0))e^{-wt}$$

Taking the limit as $t \rightarrow \infty$ then we have $0 \leq g(t) \leq \frac{v}{w}$. Hence the system is bounded in Ω .

Lemma 2. The solutions of the system (1)–(4) are non-negative for all $t \geq 0$.

Proof. First, we consider Eq. (1) of the system given as

$$\frac{dG}{dt} = r_1G + \varphi_1FG - \varphi_2PG + \varphi_3T \tag{S1}$$

Applying the condition of positivity in Eq. (S1), we can rewrite the equation as

$$\begin{aligned} \frac{dG}{dt} &\geq (r_1 + \varphi_1 F - \varphi_2 P)G \\ \Rightarrow \frac{dG}{G} &\geq A_1 dt, \text{ where } A_1 = r_1 + \varphi_1 F - \varphi_2 P \\ \Rightarrow \ln G &\geq A_1 t + \ln c_1, \text{ where } c_1 \text{ is an integrating constant.} \\ \therefore G(t) &\geq c_1 e^{A_1 t} \end{aligned} \tag{S2}$$

Applying the initial condition i.e. when $t = 0$, then Eq. (S2) becomes $G(0) = G_0 > 0$ which implies $G(0) = G_0 \geq c_1$. Now, using the value of c_1 in Eq. (S2), we have $G(t) \geq G_0 e^{A_1 t}$. Therefore, $G(t) > 0$ when $t \rightarrow \infty$.

$\therefore G(t)$ is positive for all $t \geq 0$.

Similarly, we obtained $T(t) > 0, P(t) \geq 0, F(t) \geq 0 \forall t \geq 0$ from Eqs. (2)–(4).

Hence, the lemma is completed with $G(t) > 0, T(t) > 0, P(t) \geq 0, F(t) \geq 0 \forall t \geq 0$.

Appendix C

Equilibrium points

By setting $\frac{dG}{dt} = \frac{dT}{dt} = \frac{dP}{dt} = \frac{dF}{dt} = 0$ in the system (1)–(4), two equilibrium points are obtained as

- (i) $E_G(G, T, P, F) = E_G(G^*, T^*, P^*, F^*)$ is the co-existing equilibrium point and
- (ii) $E_F(G, T, P, F) = E_F(\bar{G}, \bar{T}, \bar{P}, 0)$ is the fish population free equilibrium point.

where $G^* \approx \frac{\varphi_3 T^*}{\varphi_2 P^* - r_1 - \varphi_1 F^*}, T^* \approx \frac{r_1}{\sigma_1 \varphi_3} (r_2 - \sigma_1 P^*), P^* \approx \frac{\mu_3 \sigma_2 F^*}{2\sigma_1 (\frac{\mu_2 r_1}{\varphi_3} - \frac{r_3}{K_1})} - \frac{a-1}{2} - \frac{r_2}{2\sigma_1},$

$$F^* \approx \frac{a\eta_1 K_1 (\sigma_1 r_3 K_1 + r_2 \mu_4 K_1)}{4r_3 r_4}, \bar{G} \approx \frac{\varphi_3 \bar{T}}{\varphi_2 \bar{P} - r_1}, \bar{C} \approx \frac{\mu_1}{a\mu_4 \varphi_3} + \frac{r_2 r_3 \varphi_2}{\mu_4 \varphi_3 \sigma_2} - \frac{\mu_1 r_2}{ar_1 \mu_4 \varphi_3},$$

$$\bar{P} \approx \frac{r_2}{\sigma_2} + \frac{\varphi_3 r_1 r_3 \sigma_2 \bar{T} + 2a r_1 \varphi_2 r_2}{\sigma_2 (\mu_1 \varphi_2 - 2a r_1 \varphi_2 - a r_1 r_3 \varphi_2)}.$$

Appendix D

The stability analysis has been performed to illustrate the nature of the model (1)–(4) at each equilibrium point [42,48–50].

Stability analysis

Let's consider the system (1)–(4) in vector form to perform the stability analysis at the equilibrium points, then the vector form is given as

$$\begin{cases} \dot{x}(t) = g(\bar{x}, t) \\ x(0) = x_0 \end{cases} \tag{S3}$$

where $g = (g_1, g_2, g_3, g_4)$ and $\bar{x} = (x_1, x_2, x_3, x_4) = (G(t), T(t), P(t), F(t))$ with

$$\begin{aligned} g_1(\bar{x}, t) &= r_1 G + \varphi_1 FG - \varphi_2 PG + \varphi_3 T \\ g_2(\bar{x}, t) &= r_2 T + \sigma_1 GT - \sigma_2 PT \\ g_3(\bar{x}, t) &= r_3 P \left(1 - \frac{P}{K_1}\right) + \frac{\mu_1 P}{a+G} - \mu_2 TP - \mu_3 FP - \mu_4 GP \\ g_4(\bar{x}, t) &= r_4 F \left(1 - \frac{F}{K_2}\right) + \eta_1 PF - \frac{\eta_2 F}{a+G} - \eta_3 TF \end{aligned}$$

Therefore, the Jacobian matrix is obtained after evaluating Eq. (S3) that takes the following form

$$J_E = \begin{bmatrix} r_1 + \varphi_1 F - \varphi_2 P & \varphi_3 & -\varphi_2 G & \varphi_1 G \\ \sigma_1 T & r_2 + \sigma_1 G - \sigma_2 P & -\sigma_2 T & 0 \\ -\frac{\mu_1 P}{(a+G)^2} - \mu_4 P & -\mu_2 P & r_3 - \frac{2r_3 P}{K_1} + \frac{\mu_1}{a+G} - \mu_2 T - \mu_3 F - \mu_4 G & -\mu_3 P \\ \frac{\eta_2 F}{(a+G)^2} & -\eta_3 F & \eta_1 F & r_4 - \frac{2r_4 F}{K_2} + \eta_1 P - \frac{\eta_2}{a+G} - \eta_3 T \end{bmatrix} \tag{S4}$$

Theorem 1. The co-existing equilibrium point of the model (1)–(4) is stable or a saddle point under some conditions, otherwise unstable.

Proof. At the co-existing equilibrium point $E_G(G, T, P, F) = E_G(G^*, T^*, P^*, F^*)$, the Jacobian matrix (S4) becomes

$$J_{E_G} = \begin{bmatrix} r_1 + \varphi_1 F^* - \varphi_2 P^* & \varphi_3 & -\varphi_2 G^* & \varphi_1 G^* \\ \sigma_1 T^* & r_2 + \sigma_1 G^* - \sigma_2 P^* & -\sigma_2 T^* & 0 \\ -\frac{\mu_1 P^*}{(a+G^*)^2} - \mu_4 P^* & -\mu_2 P^* & a_{33} & -\mu_3 P^* \\ \frac{\eta_2 F^*}{(a+G^*)^2} & -\eta_3 F^* & \eta_1 F^* & a_{44} \end{bmatrix} \tag{S5}$$

$$= \begin{bmatrix} -\frac{\mu_1 P^*}{(a+G^*)^2} - \mu_4 P^* & -\mu_2 P^* & a_{33} & -\mu_3 P^* \\ 0 & -b_{22} & -b_{23} & 0 \\ 0 & 0 & -c_{33} & b_{34} \\ 0 & 0 & 0 & -d_{44} \end{bmatrix}$$

The characteristic equation of the matrix (S5) takes the form as

$$|J_{E_G} - \lambda I| = \begin{vmatrix} -\frac{\mu_1 P^*}{(a+G^*)^2} - \mu_4 P^* - \lambda & -\mu_2 P^* & a_{33} & -\mu_3 P^* \\ 0 & -b_{22} - \lambda & -b_{23} & 0 \\ 0 & 0 & -c_{33} - \lambda & b_{34} \\ 0 & 0 & 0 & -d_{44} - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \left(-\frac{\mu_1 P^*}{(a+G^*)^2} - \mu_4 P^* - \lambda\right) (-b_{22} - \lambda) (-c_{33} - \lambda) (-d_{44} - \lambda) = 0$$

Hence the eigenvalues are

$$\lambda_1 = -\left(\frac{\mu_1 P^*}{(a+G^*)^2} + \mu_4 P^*\right), \lambda_2 = \frac{(\mu_1 \sigma_1 - 2a r_2 \mu_4) G^* - (\mu_4 - \mu_1 r_2)}{\mu_1 + 2a \mu_4 G^*}$$

$$\lambda_3 = \frac{2a r_3 \varphi_2 G^*}{\mu_1 + 2a \mu_4 G^*} + \frac{\sigma_2 \varphi_3 \mu_1 G^*}{(\mu_4 - \mu_1 r_2)} - \varphi_2 G^*, \lambda_4 = -\left\{\frac{\eta_2 \mu_3 F^*}{\mu_1} + \frac{\eta_1 \mu_2 r_2 F^*}{\varphi_2 \mu_4} + \frac{\eta_1 \mu_1 \mu_2 r_2 P^*}{\mu_4} + \frac{\mu_1 \mu_2 r_2 r_4}{\mu_4} - r_4\right\}.$$

It is clear that two eigenvalues λ_1 and λ_4 are negative. Therefore, the co-existing equilibrium point is stable if $(\mu_4 - \mu_1 r_2) > (\mu_1 \sigma_1 - 2a r_2 \mu_4) G^*$ and $\varphi_2 G^* > \frac{2a r_3 \varphi_2 G^*}{\mu_1 + 2a \mu_4 G^*} + \frac{\sigma_2 \varphi_3 \mu_1 G^*}{(\mu_4 - \mu_1 r_2)}$, or it is a saddle point if $(\mu_4 - \mu_1 r_2) < (\mu_1 \sigma_1 - 2a r_2 \mu_4) G^*$ and $\varphi_2 G^* < \frac{2a r_3 \varphi_2 G^*}{\mu_1 + 2a \mu_4 G^*} + \frac{\sigma_2 \varphi_3 \mu_1 G^*}{(\mu_4 - \mu_1 r_2)}$, otherwise unstable. where, $a_{33} = r_3 - \frac{2r_3}{K_1} P + \frac{\mu_1}{a+G} - \mu_2 T - \mu_3 F - \mu_4 G$, $a_{44} = r_4 - \frac{2r_4}{K_2} F + \eta_1 P^* - \frac{\eta_2}{a+G^*} - \eta_3 T^*$, $b_{22} = \frac{(\mu_4 - \mu_1 r_2) - (\mu_1 \sigma_1 - 2a r_2 \mu_4) G^*}{\mu_1 + 2a \mu_4 G^*}$, $b_{23} = \sigma_2 T^* + \frac{4a \sigma_1 r_3 T^* G^*}{\mu_1 + 2a \mu_4 G^*}$, $b_{32} = \varphi_3 - \frac{2a r_1 \mu_2 G^*}{\mu_1 + 2a \mu_4 G^*}$, $b_{33} = \varphi_2 G^* + \frac{2a r_3 \varphi_2 G^*}{\mu_1 + 2a \mu_4 G^*}$, $b_{34} = \varphi_1 G^* - \frac{2a r_1 \mu_3 G^*}{\mu_1 + 2a \mu_4 G^*}$, $b_{42} = \eta_3 F^* + \frac{\eta_2 \mu_2 F^*}{\mu_1 + 2a \mu_4 G^*}$, $b_{43} = \eta_1 F^* + \frac{r_3 \eta_2 F^* - \eta_2 \mu_4 F^* G^*}{\mu_1 P^* + 2a \mu_4 P^* G^*}$, $b_{44} = r_4 + \eta_1 P^* - \frac{\eta_2 \mu_3 F^*}{\mu_1 + 2a \mu_4 G^*}$, $c_{33} = \varphi_2 G^* - \frac{2a r_3 \varphi_2 G^*}{\mu_1 + 2a \mu_4 G^*} - \frac{\sigma_2 \varphi_3 \mu_1 G^*}{(\mu_4 - \mu_1 r_2)}$, $c_{43} = \frac{\eta_1 \mu_4 F^* - \eta_1 \mu_2 r_2 F^*}{\mu_4 - \mu_1 r_2}$, $d_{44} = \frac{\eta_2 \mu_3 F^*}{\mu_1} + \frac{\eta_1 \mu_2 r_2 F^*}{\varphi_2 \mu_4} + \frac{\eta_1 \mu_1 \mu_2 r_2 P^*}{\mu_4} + \frac{\mu_1 \mu_2 r_2 r_4}{\mu_4} - r_4$.

Theorem 2. The fish population free equilibrium point of the model (1)–(4) is stable or a saddle point under some conditions, otherwise unstable.

Proof. At the fish population free equilibrium point $E_F(G, T, P, 0) = E_F(\bar{G}, \bar{T}, \bar{P}, 0)$, the Jacobian matrix (S4) becomes

$$J_{E_F} = \begin{bmatrix} r_1 - \varphi_2 \bar{P} & \varphi_3 & -\varphi_2 \bar{G} & \varphi_1 \bar{G} \\ \sigma_1 \bar{T} & r_2 + \sigma_1 \bar{G} - \sigma_2 \bar{P} & -\sigma_2 \bar{T} & 0 \\ -\frac{\mu_1 \bar{P}}{(a+\bar{G})^2} - \mu_4 \bar{P} & -\mu_2 \bar{P} & a_{33} & -\mu_3 \bar{P} \\ 0 & 0 & 0 & r_4 + \eta_1 \bar{P} - \frac{\eta_2}{a+\bar{G}} - \eta_3 \bar{T} \end{bmatrix} \tag{S6}$$

$$= \begin{bmatrix} -\frac{\mu_1 \bar{P}}{(a+\bar{G})^2} - \mu_4 \bar{P} & -\mu_2 \bar{P} & a_{33} & -\mu_3 \bar{P} \\ 0 & b_{22} & -b_{23} & -\frac{2a \sigma_1 \mu_3 \bar{T} \bar{G}}{\mu_1 + 2a \mu_4 \bar{G}} \\ 0 & 0 & -c_{33} & c_{34} \\ 0 & 0 & 0 & r_4 + \eta_1 \bar{P} - \frac{\eta_2}{a+\bar{G}} - \eta_3 \bar{T} \end{bmatrix}$$

The characteristic equation of the above matrix (S6) takes the following form

$$|J_{E_F} - \lambda I| = \begin{vmatrix} -\frac{\mu_1 \bar{P}}{(a+\bar{G})^2} - \mu_4 \bar{P} - \lambda & -\mu_2 \bar{P} & a_{33} & -\mu_3 \bar{P} \\ 0 & b_{22} - \lambda & -b_{23} & -\frac{2a \sigma_1 \mu_3 \bar{T} \bar{G}}{\mu_1 + 2a \mu_4 \bar{G}} \\ 0 & 0 & -c_{33} - \lambda & c_{34} \\ 0 & 0 & 0 & r_4 + \eta_1 \bar{P} - \frac{\eta_2}{a+\bar{G}} - \eta_3 \bar{T} - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \left(-\frac{\mu_1 \bar{P}}{(a+\bar{G})^2} - \mu_4 \bar{P} - \lambda\right) (b_{22} - \lambda) (-c_{33} - \lambda) \left(r_4 + \eta_1 \bar{P} - \frac{\eta_2}{a+\bar{G}} - \eta_3 \bar{T} - \lambda\right) = 0$$

Hence the eigenvalues are

$$\lambda_1 = -\left\{\frac{\mu_1 \bar{P}}{(a+\bar{G})^2} + \mu_4 \bar{P}\right\}, \lambda_2 = r_2 + \sigma_1 \bar{G} - \sigma_2 \bar{P} - \frac{2a \sigma_1 \mu_2 \bar{T} \bar{G}}{\mu_1 + 2a \mu_4 \bar{G}},$$

$$\lambda_3 = -\left\{\varphi_2 \bar{G} + \frac{2a r_3 \bar{G}}{\mu_1 + 2a \mu_4 \bar{G}} \left(\varphi_2 - \frac{r_2}{\bar{P}}\right) - \frac{\sigma_2 \varphi_2 \bar{T}}{r_2 + \sigma_1 \bar{G} - \sigma_1 \bar{P}}\right\}, \lambda_4 = r_4 + \eta_1 \bar{P} - \frac{\eta_2}{a+\bar{G}} - \eta_3 \bar{T}.$$

Hence the equilibrium point is stable when $r_2 + \sigma_1 \bar{G} < \sigma_2 \bar{P} + \frac{2a\sigma_1\mu_2\bar{T}\bar{G}}{\mu_1+2a\mu_4\bar{G}}$, $\varphi_2\bar{G} + \frac{2a\sigma_3\bar{G}}{\mu_1+2a\mu_4\bar{G}}(\varphi_2 - \frac{r_2}{\bar{P}}) > \frac{\sigma_2\varphi_2\bar{T}}{r_2+\sigma_1\bar{G}-\sigma_1\bar{P}}$ and $r_4 + \eta_1\bar{P} < \frac{\eta_2}{a+\bar{G}} + \eta_3\bar{T}$, or it will be a saddle point if $r_2 + \sigma_1\bar{G} > \sigma_2\bar{P} + \frac{2a\sigma_1\mu_2\bar{T}\bar{G}}{\mu_1+2a\mu_4\bar{G}}$, $\varphi_2\bar{G} + \frac{2a\sigma_3\bar{G}}{\mu_1+2a\mu_4\bar{G}}(\varphi_2 - \frac{r_2}{\bar{P}}) > \frac{\sigma_2\varphi_2\bar{T}}{r_2+\sigma_1\bar{G}-\sigma_1\bar{P}}$ and $r_4 + \eta_1\bar{P} > \frac{\eta_2}{a+\bar{G}} + \eta_3\bar{T}$, otherwise it will be unstable. where, $a_{33} = r_3 - \frac{2r_3}{K_1}\bar{P} + \frac{\mu_1}{a+\bar{G}} - \mu_2\bar{T} - \mu_4\bar{G}$, $b_{22} = r_2 + \sigma_1\bar{G} - \sigma_2\bar{P} - \frac{2a\sigma_1\mu_2\bar{T}\bar{G}}{\mu_1+2a\mu_4\bar{G}}$, $b_{23} = -\sigma_2\bar{C} - \frac{2a\sigma_1r_3\bar{T}\bar{G}}{\bar{P}(\mu_1+2a\mu_4\bar{G})} - \frac{4a\sigma_1r_3\bar{T}\bar{G}}{K_1(\mu_1+2a\mu_4\bar{G})}$, $b_{32} = \varphi_3 - \frac{2a\mu_2\bar{G}}{\mu_1+2a\mu_4\bar{G}}(r_2 - \varphi_2\bar{P})$, $b_{33} = \varphi_2\bar{G} - \frac{2a\sigma_3\bar{G}}{\mu_1\bar{P}+2a\mu_4\bar{P}\bar{G}}(r_2 - \varphi_2\bar{P}) + \frac{4a\sigma_3\bar{G}}{K_1(\mu_1+2a\mu_4\bar{G})}(r_2 - \varphi_2\bar{P})$, $b_{34} = \varphi_1\bar{G} - \frac{2a\mu_3\bar{G}}{\mu_1+2a\mu_4\bar{G}}(r_2 - \varphi_2\bar{P})$, $c_{33} = \varphi_2\bar{G} + \frac{2a\sigma_3\bar{G}}{\mu_1+2a\mu_4\bar{G}}(\varphi_2 - \frac{r_2}{\bar{P}}) - \frac{\sigma_2\varphi_2\bar{T}}{r_2+\sigma_1\bar{G}-\sigma_1\bar{P}}$, $c_{34} = \varphi_1\bar{G} - \frac{2a\sigma_3\bar{G}}{\mu_1+2a\mu_4\bar{G}}(r_2 - \varphi_2\bar{P}) + \frac{2a\sigma_1\varphi_2\mu_3\bar{T}\bar{G}}{r_2\mu_1+\sigma_1\mu_1\bar{G}-\sigma_1\mu_1\bar{P}}$.

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